Combinational Circuit Example (4A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Boolean functions of **a** and **b**

What are these?

$\bar{a}\cdot\bar{b}$ $\bar{a}\cdot b$ $a\cdot\bar{b}$ $a\cdot b$

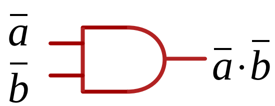
Consider them as a function

$f_0(a,b) = \bar{a}\cdot\bar{b}$ $f_1(a,b) = \bar{a}\cdot b$ $f_2(a,b) = a\cdot\bar{b}$ $f_3(a,b) = a\cdot b$

Boolean Function Definition for $\mathbf{f_i} = 1$

$f_0(a,b) = \bar{a}\cdot\bar{b}$

a	b	f_0
0	0	1
0	1	0
1	0	0
1	1	0



$\bar{a}\cdot\bar{b} = 1 \Leftrightarrow$

$(\bar{a}=1) \wedge (\bar{b}=1)$

$\bar{a}\cdot\bar{b} = 1$ *when*

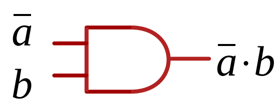
$(a=0) \wedge (b=0)$

$\bar{a}\cdot\bar{b} = 0$

for other three cases

$f_1(a,b) = \bar{a}\cdot b$

a	b	f_1
0	0	0
0	1	1
1	0	0
1	1	0



$\bar{a}\cdot b = 1 \Leftrightarrow$

$(\bar{a}=1) \wedge (b=1)$

$\bar{a}\cdot b = 1$ *when*

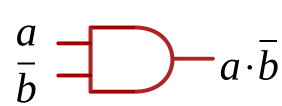
$(a=0) \wedge (b=1)$

$\bar{a}\cdot b = 0$

for other three cases

$f_2(a,b) = a\cdot\bar{b}$

a	b	f_2
0	0	0
0	1	0
1	0	1
1	1	0



$a\cdot\bar{b} = 1 \Leftrightarrow$

$(a=1) \wedge (\bar{b}=1)$

$a\cdot\bar{b} = 1$ *when*

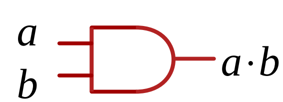
$(a=1) \wedge (b=0)$

$a\cdot\bar{b} = 0$

for other three cases

$f_3(a,b) = a\cdot b$

a	b	f_3
0	0	0
0	1	0
1	0	0
1	1	1



$a\cdot b = 1 \Leftrightarrow$

$(a=1) \wedge (b=1)$

$a\cdot b = 1$ *when*

$(a=1) \wedge (b=1)$

$a\cdot b = 0$

for other three cases

Alternative Boolean Function Definition (1)

$$f_0(a,b) = \bar{a}\cdot\bar{b}$$

a	b	f_0
0	0	1
0	1	0
1	0	0
1	1	0

$$(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$$

a	b	$(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$
0	0	$(0+\bar{0})\cdot(\bar{0}+0)\cdot(\bar{0}+\bar{0})=1$
0	1	$(0+\bar{1})\cdot(\bar{0}+1)\cdot(\bar{0}+\bar{1})=0$
1	0	$(1+\bar{0})\cdot(\bar{1}+0)\cdot(\bar{1}+\bar{0})=0$
1	1	$(1+\bar{1})\cdot(\bar{1}+1)\cdot(\bar{1}+\bar{1})=0$

$$f_0(a,b) = \bar{a}\cdot\bar{b} = (a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$$

$$f_1(a,b) = \bar{a}\cdot b$$

a	b	f_1
0	0	0
0	1	1
1	0	0
1	1	0

$$(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$$

a	b	$(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$
0	0	$(0+0)\cdot(\bar{0}+0)\cdot(\bar{0}+\bar{0})=0$
0	1	$(0+1)\cdot(\bar{0}+1)\cdot(\bar{0}+\bar{1})=1$
1	0	$(1+0)\cdot(\bar{1}+0)\cdot(\bar{1}+\bar{0})=0$
1	1	$(1+1)\cdot(\bar{1}+1)\cdot(\bar{1}+\bar{1})=0$

$$f_1(a,b) = \bar{a}\cdot b = (a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$$

$$f_2(a,b) = a\cdot\bar{b}$$

a	b	f_2
0	0	0
0	1	0
1	0	1
1	1	0

$$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})$$

a	b	$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})$
0	0	$(0+0)\cdot(0+\bar{0})\cdot(\bar{0}+\bar{0})=0$
0	1	$(0+1)\cdot(0+\bar{1})\cdot(\bar{0}+\bar{1})=0$
1	0	$(1+0)\cdot(1+\bar{0})\cdot(\bar{1}+\bar{0})=1$
1	1	$(1+1)\cdot(1+\bar{1})\cdot(\bar{1}+\bar{1})=0$

$$f_2(a,b) = a\cdot\bar{b} = (a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})$$

$$f_3(a,b) = a\cdot b$$

a	b	f_3
0	0	0
0	1	0
1	0	0
1	1	1

$$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)$$

a	b	$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)$
0	0	$(0+0)\cdot(0+\bar{0})\cdot(\bar{0}+0)=0$
0	1	$(0+1)\cdot(0+\bar{1})\cdot(\bar{0}+1)=0$
1	0	$(1+0)\cdot(1+\bar{0})\cdot(\bar{1}+0)=0$
1	1	$(1+1)\cdot(1+\bar{1})\cdot(\bar{1}+1)=1$

$$f_3(a,b) = a\cdot b = (a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)$$

$f_0(a,b) =$ $f_1(a,b) =$ $f_2(a,b) =$ $f_3(a,b) =$

Alternative Boolean Function Definition (2)

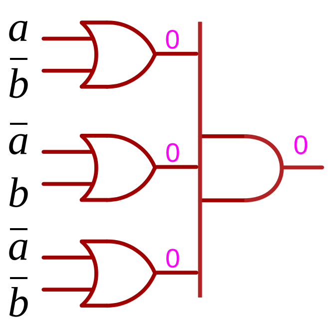
$f_0(a,b) = \bar{a}\cdot\bar{b}$	$f_1(a,b) = \bar{a}\cdot b$	$f_2(a,b) = a\cdot\bar{b}$	$f_3(a,b) = a\cdot b$	$\Rightarrow$ 1
$\bar{a}\cdot\bar{b} = 1 \Leftrightarrow$ $(\bar{a}=1) \wedge (\bar{b}=1)$	$\bar{a}\cdot b = 1 \Leftrightarrow$ $(\bar{a}=1) \wedge (b=1)$	$a\cdot\bar{b} = 1 \Leftrightarrow$ $(a=1) \wedge (\bar{b}=1)$	$a\cdot b = 1 \Leftrightarrow$ $(a=1) \wedge (b=1)$	
$\bar{a}\cdot\bar{b} = 1$ *when* $(a=0) \wedge (b=0)$	$\bar{a}\cdot b = 1$ *when* $(a=0) \wedge (b=1)$	$a\cdot\bar{b} = 1$ *when* $(a=1) \wedge (b=0)$	$a\cdot b = 1$ *when* $(a=1) \wedge (b=1)$	

$f_0(a,b) =$ $(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$	$f_1(a,b) =$ $(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$	$f_2(a,b) =$ $(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})$	$f_3(a,b) =$ $(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)$	$\Rightarrow$ 0
$(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b}) = 0 \Leftrightarrow$ $(a+\bar{b}=0)\vee(\bar{a}+b=0)\vee(\bar{a}+\bar{b}=0)$	$(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})=0 \Leftrightarrow$ $(a+b=0)\vee(\bar{a}+b=0)\vee(\bar{a}+\bar{b}=0)$	$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})=0 \Leftrightarrow$ $(a+b=0)\vee(a+\bar{b}=0)\vee(\bar{a}+\bar{b}=0)$	$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)=0 \Leftrightarrow$ $(a+b=0)\vee(a+\bar{b}=0)\vee(\bar{a}+b=0)$	
$\bar{a}\cdot\bar{b} = 0$ *when* $[(a=0)\wedge(b=1)]\vee$ $[(a=1)\wedge(b=0)]\vee$ $[(a=1)\wedge(b=1)]$	$\bar{a}\cdot b = 0$ *when* $[(a=0)\wedge(b=0)]\vee$ $[(a=1)\wedge(b=0)]\vee$ $[(a=1)\wedge(b=1)]$	$a\cdot\bar{b} = 0$ $[(a=0)\wedge(b=0)]\vee$ $[(a=0)\wedge(b=1)]\vee$ $[(a=1)\wedge(b=1)]$	$a\cdot b = 0$ $[(a=0)\wedge(b=0)]\vee$ $[(a=0)\wedge(b=1)]\vee$ $[(a=1)\wedge(b=0)]$	

Boolean Function Definition for $\mathbf{f_i} = 0$

$f_0(a,b) =$
$(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$

a	b	f_1
0	0	1
0	1	0
1	0	0
1	1	0



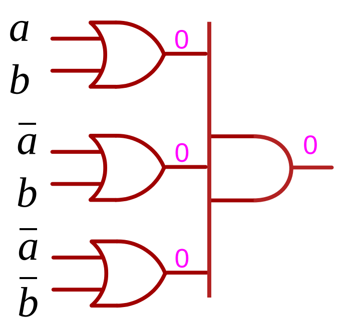
$(a+\bar{b})\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b}) = 0$

$(a+\bar{b}=0)\vee(\bar{a}+b=0)\vee(\bar{a}+\bar{b}=0)$

$(a=0\wedge b=1)\vee$
$(a=1\wedge b=0)\vee$
$(a=1\wedge b=1)$

$f_1(a,b) =$
$(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b})$

a	b	f_2
0	0	0
0	1	1
1	0	0
1	1	0



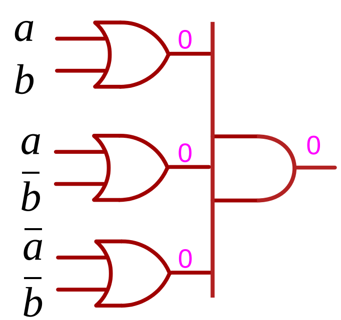
$(a+b)\cdot(\bar{a}+b)\cdot(\bar{a}+\bar{b}) = 0$

$(a+b=0)\vee(\bar{a}+b=0)\vee(\bar{a}+\bar{b}=0)$

$(a=0\wedge b=0)\vee$
$(a=1\wedge b=0)\vee$
$(a=1\wedge b=1)$

$f_2(a,b) =$
$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b})$

a	b	f_3
0	0	0
0	1	0
1	0	1
1	1	0



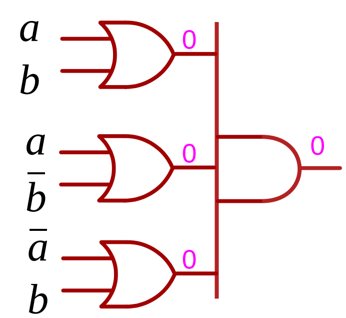
$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+\bar{b}) = 0$

$(a+b=0)\vee(a+\bar{b}=0)\vee(\bar{a}+\bar{b}=0)$

$(a=0\wedge b=0)\vee$
$(a=0\wedge b=1)\vee$
$(a=1\wedge b=1)$

$f_3(a,b) =$
$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b)$

a	b	f_4
0	0	0
0	1	0
1	0	0
1	1	1



$(a+b)\cdot(a+\bar{b})\cdot(\bar{a}+b) = 0$

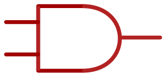
$(a+b=0)\vee(a+\bar{b}=0)\vee(\bar{a}+b=0)$

$(a=0\wedge b=0)\vee$
$(a=0\wedge b=1)\vee$
$(a=1\wedge b=0)$

Logic gates in terms of $\mathbf{f_i}$

Use $\mathbf{f_0}$, $\mathbf{f_1}$, $\mathbf{f_2}$, $\mathbf{f_3}$ to express logic gates

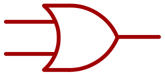
$a \cdot b$



a	b	•
0	0	0
0	1	0
1	0	0
1	1	1

$$a \cdot b = f_3$$

$a + b$

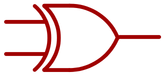


a	b	+
0	0	0
0	1	1
1	0	1
1	1	1

$$a + b = f_1 + f_2 + f_3$$

$$a + b = a \oplus b + a \cdot b$$

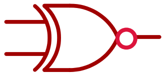
$a \oplus b$



a	b	⊕
0	0	0
0	1	1
1	0	1
1	1	0

$$a \oplus b = f_1 + f_2$$

$\overline{a \oplus b}$



a	b	⊕
0	0	1
0	1	0
1	0	0
1	1	1

$$\overline{a \oplus b} = f_0 + f_3$$

XOR gate in terms of $\mathbf{f_i}$

Use $\mathbf{f_1}$, $\mathbf{f_2}$ to express an XOR logic gate

$a \oplus b$

a	b	⊕
0	0	0
0	1	1
1	0	1
1	1	0

=

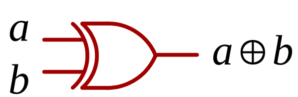
$f_1(a,b) = \bar{a} \cdot b$

a	b	f_1
0	0	0
0	1	1
1	0	0
1	1	0

+

$f_2(a,b) = a \cdot \bar{b}$

a	b	f_2
0	0	0
0	1	0
1	0	1
1	1	0



defined by the truth table

$a \oplus b = 1 \Leftrightarrow$

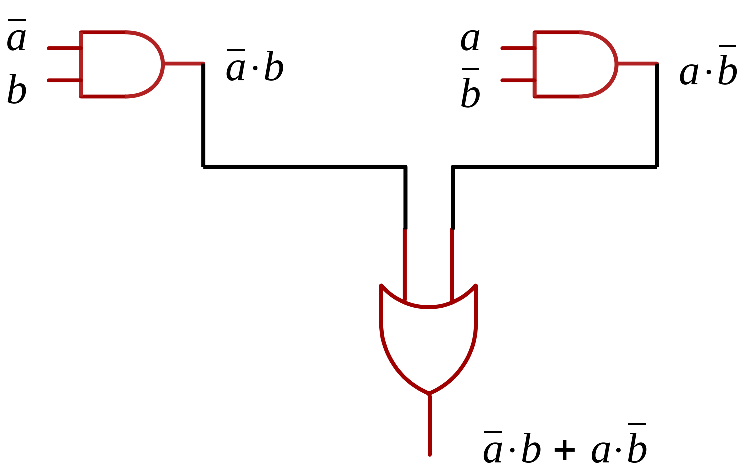
$(a=1 \;\wedge\; b=0) \;\vee\; (a=0 \;\wedge\; b=1)$

$\bar{a} \cdot b + a \cdot \bar{b} = 1 \Leftrightarrow$

$(\bar{a} \cdot b = 1) \;\vee\; (a \cdot \bar{b} = 1) \Leftrightarrow$

$(a=0 \wedge b=1) \;\vee\; (a=1 \wedge b=0)$

combined by the minterms



XNOR gate in terms of $\mathbf{f_i}$

Use $\mathbf{f_0}$, $\mathbf{f_3}$ to express an XNOR logic gate

$a \oplus b$ $\qquad$ $f_0(a,b) = \bar{a} \cdot \bar{b}$ $\qquad$ $f_3(a,b) = a \cdot b$

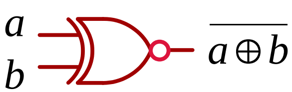
a	b	⊕
0	0	0
0	1	1
1	0	1
1	1	0

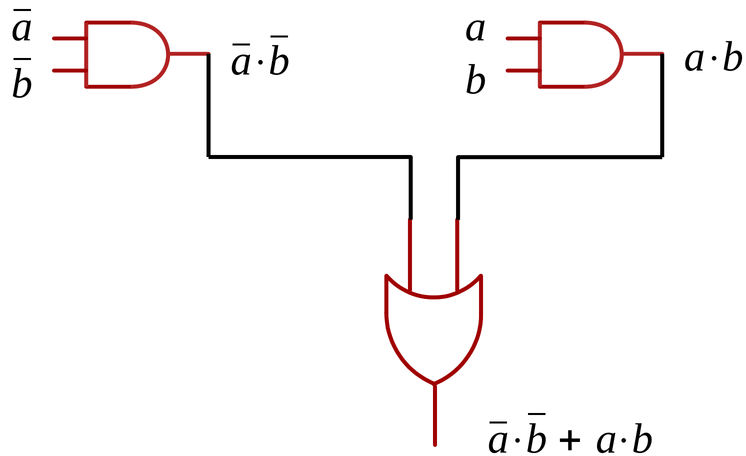
=

a	b	f_0
0	0	1
0	1	0
1	0	0
1	1	0

+

a	b	f_3
0	0	0
0	1	0
1	0	0
1	1	1





defined by the truth table

$\overline{a \oplus b} = 1 \Leftrightarrow$

$(a=0 \ \wedge \ b=0) \vee (a=1 \ \wedge \ b=1)$

$\bar{a} \cdot \bar{b} + a \cdot b = 1 \Leftrightarrow$

$(\bar{a} \cdot \bar{b} = 1) \vee (a \cdot b = 1) \Leftrightarrow$

$(a=0 \wedge b=0) \vee (a=1 \wedge b=1)$

combined by the minterms

OR gate in terms of $\mathbf{f_i}$

Use $\mathbf{f_1}$, $\mathbf{f_2}$, $\mathbf{f_3}$ to express an OR logic gate

$$a+b \quad = \quad f_1(a,b) = \bar{a}\cdot b \quad + \quad f_2(a,b) = a\cdot\bar{b} \quad + \quad f_3(a,b) = a\cdot b$$

a	b	+
0	0	0
0	1	1
1	0	1
1	1	1

=

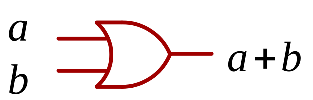
a	b	f_1
0	0	0
0	1	1
1	0	0
1	1	0

+

a	b	f_2
0	0	0
0	1	0
1	0	1
1	1	0

+

a	b	f_3
0	0	0
0	1	0
1	0	0
1	1	1



defined by the truth table

$a+b = 1 \Leftrightarrow$

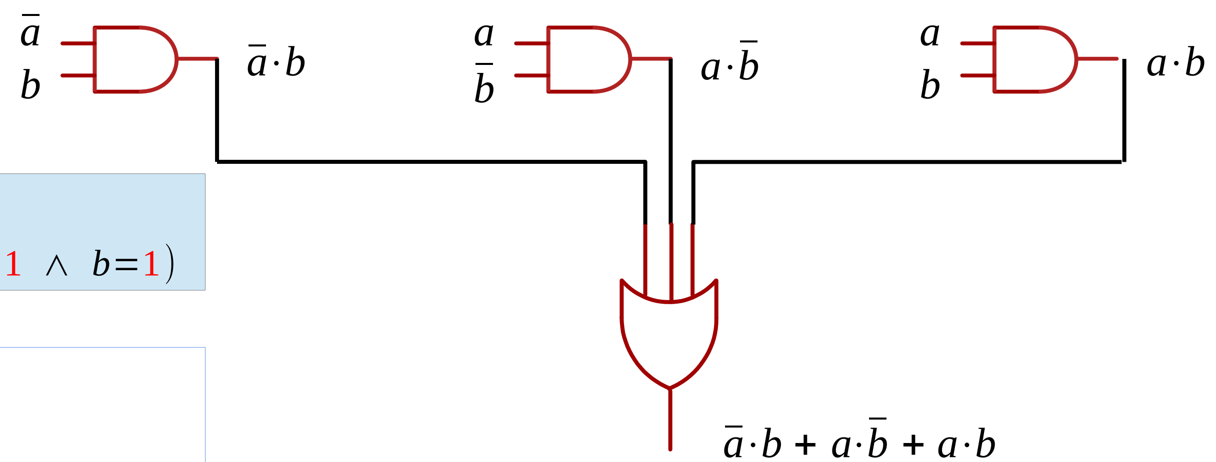
$(a=0 \;\wedge\; b=1) \;\vee\; (a=1 \;\wedge\; b=0) \;\vee\; (a=1 \;\wedge\; b=1)$

$\bar{a}\cdot b + a\cdot\bar{b} + a\cdot b \;=\; 1 \Leftrightarrow$

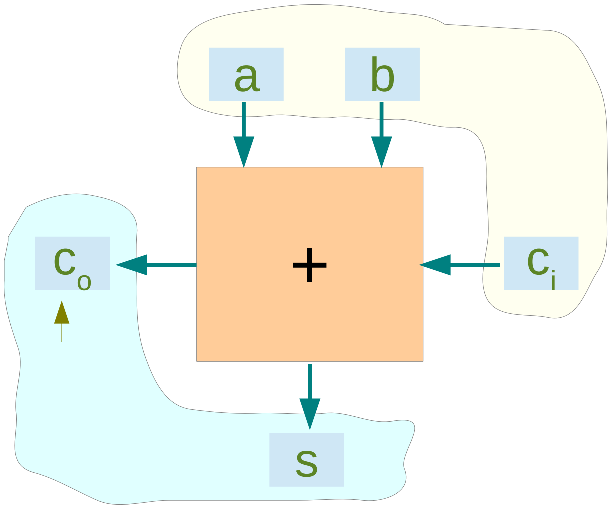
$(\bar{a}\cdot b=1) \;\vee\; (a\cdot\bar{b}=1) \;\vee\; (a\cdot b=1) \Leftrightarrow$

$(a=0 \wedge b=1) \;\vee\; (a=1 \wedge b=0) \;\vee\; (a=1 \wedge b=1)$

combined by the minterms



Adding three 1-bit numbers



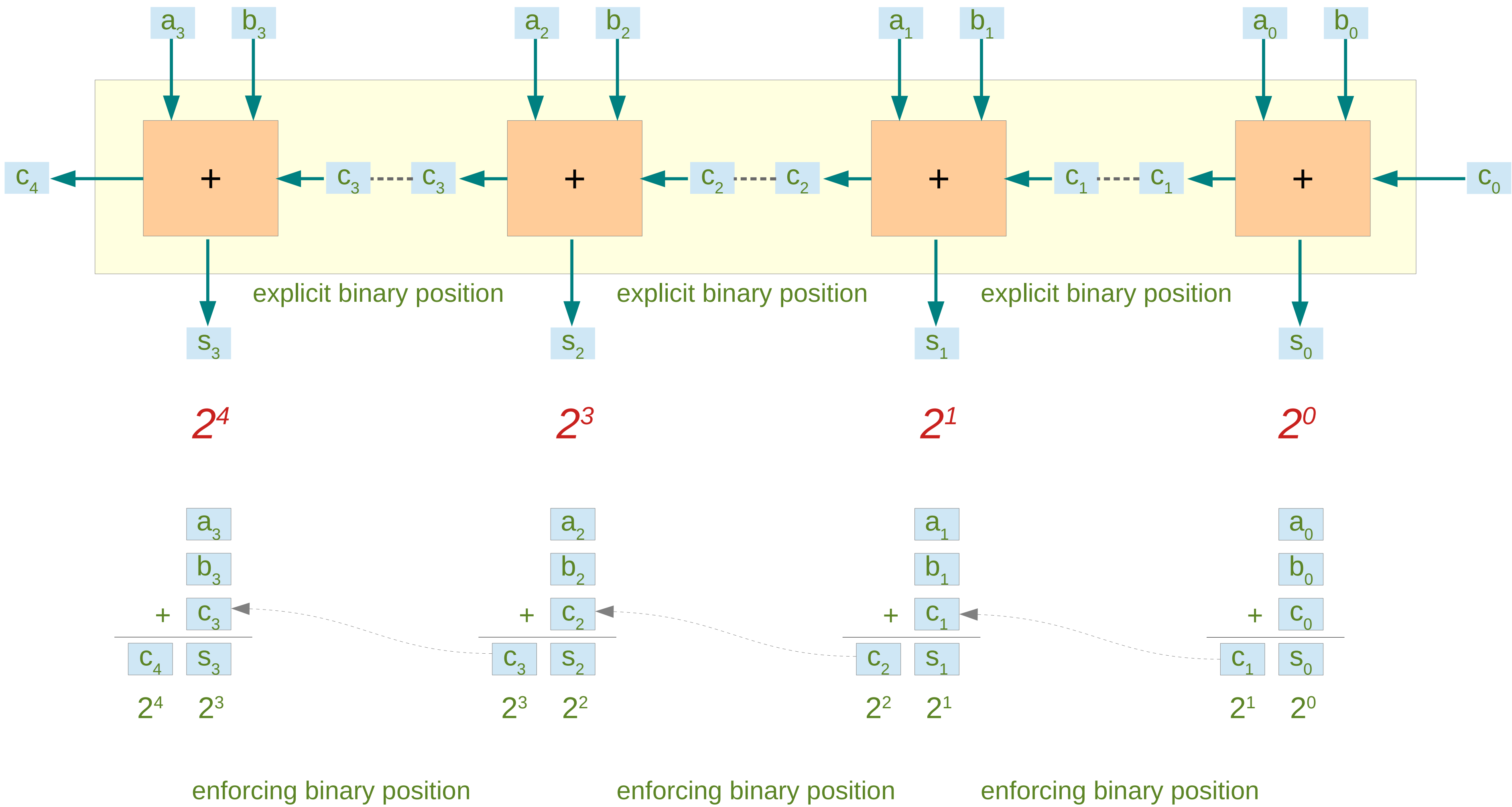
no binary position assumed

0	0	1	1
0	1	0	1
+ 0	+ 0	+ 0	+ 0
0 0	0 1	0 1	1 0

a	0	0	1	1
b	0	1	0	1
+ c_i	+ 1	+ 1	+ 1	+ 1
c_o s	0 1	1 0	1 0	1 1

implicit binary position

A 4-bit adder


a3 b3 a2 b2 a1 b1 a0 b0
c4 + c3 c3 + c2 c2 + c1 c1 + c0
s3 s2 s1 s0
explicit binary position
explicit binary position
explicit binary position
2^4 2^3 2^1 2^0
a3 b3 + c3 c4 s3 2^4 2^3
a2 b2 + c2 c3 s2 2^3 2^2
a1 b1 + c1 c2 s1 2^2 2^1
a0 b0 + c0 c1 s0 2^1 2^0
enforcing binary position
enforcing binary position
enforcing binary position

K-maps for output c_o and s

a	b	c_i	c_o
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

inputs

c_o

c_i \ a b	00	01	11	10
0	0	0	1	0
1	0	1	1	1

a	b	c_i	s
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

inputs

s

c_i \ a b	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$c_o = \bar{a}\cdot b\cdot c_i + a\cdot\bar{b}\cdot c_i + a\cdot b\cdot\bar{c}_i + a\cdot b\cdot c_i$$

$$= a\cdot b + b\cdot c_i + c_i\cdot a$$ by the K-map simplification

$$s = \bar{a}\cdot\bar{b}\cdot c_i + \bar{a}\cdot b\cdot\bar{c}_i + a\cdot\bar{b}\cdot\bar{c}_i + a\cdot b\cdot c_i$$

$$c_o = \left(\bar{a}\cdot b + a\cdot\bar{b}\right)\cdot c_i + \left(a\cdot b\right)\cdot\left(\bar{c}_i + c_i\right)$$

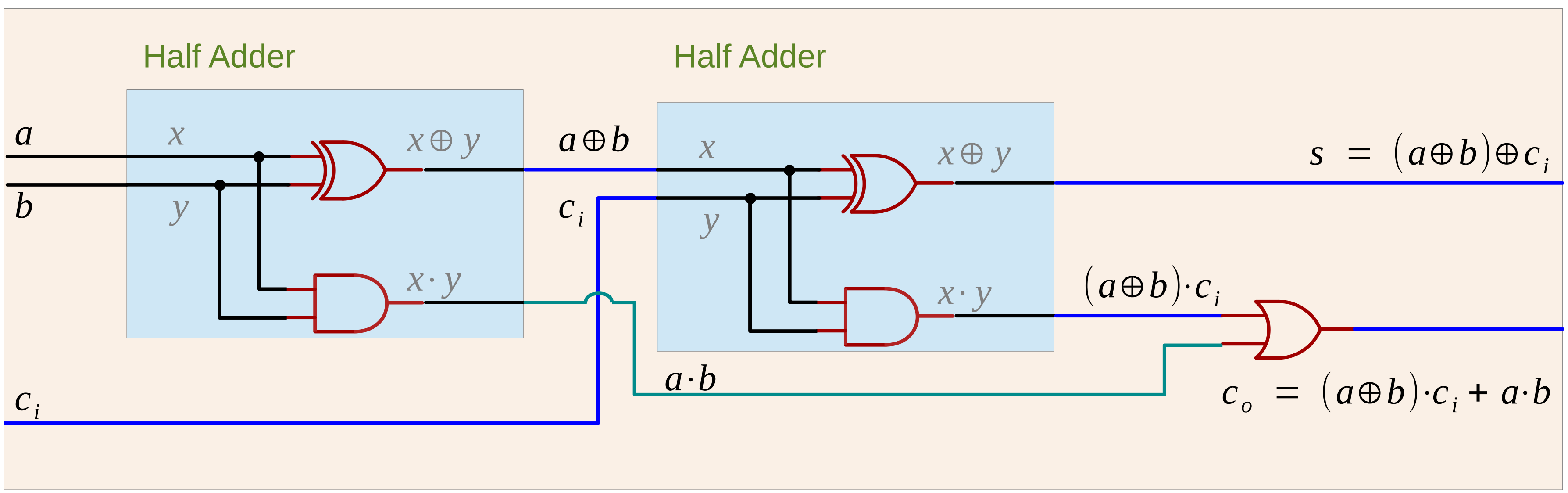
$$= \left(a\oplus b\right)\cdot c_i + a\cdot b$$ by the insight

Hand minimization for the output c_o and s

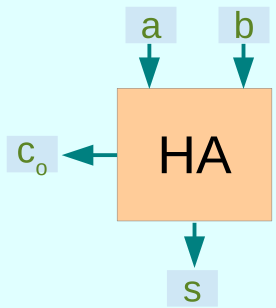
$$
\begin{aligned}
c_o &= a \cdot b + b \cdot c_i + c_i \cdot a \\
&= a \cdot b + (a + b) \cdot c_i \qquad a+b = m_1+m_2+m_3 \;\; minterms \\
&= a \cdot b + (a \cdot \bar{b} + \bar{a} \cdot b + a \cdot b) \cdot c_i \\
&= a \cdot b + a \cdot b \cdot c_i + (a \cdot \bar{b} + \bar{a} \cdot b) \cdot c_i \\
&= a \cdot b \cdot (1 + c_i) + (a \oplus b) \cdot c_i \\
&= a \cdot b + (a \oplus b) \cdot c_i
\end{aligned}
$$

$$
\begin{aligned}
s &= \bar{a} \cdot \bar{b} \cdot c_i + \bar{a} \cdot b \cdot \bar{c}_i + a \cdot \bar{b} \cdot \bar{c}_i + a \cdot b \cdot c_i \\
&= \bar{a} \cdot \bar{b} \cdot c_i + a \cdot b \cdot c_i + \bar{a} \cdot b \cdot \bar{c}_i + a \cdot \bar{b} \cdot \bar{c}_i \\
&= (\bar{a} \cdot \bar{b} + a \cdot b) \cdot c_i + (\bar{a} \cdot b + a \cdot \bar{b}) \cdot \bar{c}_i \\
&= (\overline{a \oplus b}) \cdot c_i + (a \oplus b) \cdot \bar{c}_i \\
&= (a \oplus b) \oplus c_i
\end{aligned}
$$

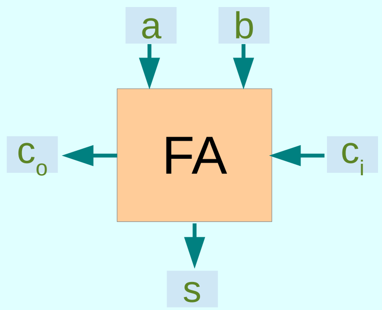
Full Adder



Full Adders and Half Adders



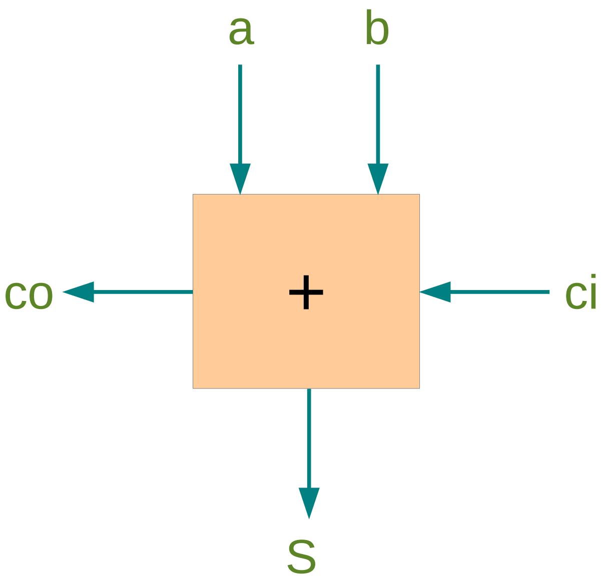
$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 1\ 0 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 0 \\ +\ 1 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 0 \\ +\ 0 \\ \hline 0\ 0 \end{array}$$



$$\begin{array}{r} 0 \\ 0 \\ +\ 0 \\ \hline 0\ 0 \end{array} \qquad \begin{array}{r} 0 \\ 1 \\ +\ 0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ 0 \\ +\ 0 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 1 \\ 1 \\ +\ 0 \\ \hline 1\ 0 \end{array}$$

$$\begin{array}{r} 0 \\ 0 \\ +\ 1 \\ \hline 0\ 1 \end{array} \qquad \begin{array}{r} 0 \\ 1 \\ +\ 1 \\ \hline 1\ 0 \end{array} \qquad \begin{array}{r} 1 \\ 0 \\ +\ 1 \\ \hline 1\ 0 \end{array} \qquad \begin{array}{r} 1 \\ 1 \\ +\ 1 \\ \hline 1\ 1 \end{array}$$

Truth Table



$$\begin{array}{rr} & \boxed{a} \\ & \boxed{b} \\ + & \boxed{ci} \\ \hline & \boxed{co} \end{array}$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

Truth Table

References

[1] http://en.wikipedia.org/