

Address-of and dereference operators

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& address-of operator

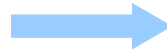
***** dereference operator

Address-of operator and dereferencing operator

address-of operator **&** : the address of a variable

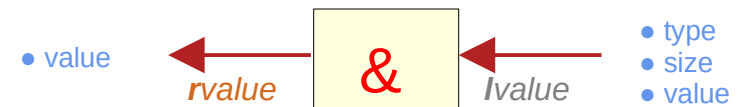
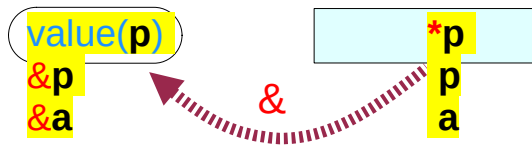
a **variable** has a memory location
whose value can be changed
by an assignment

a **variable** must be an **lvalue**



&variable returns
the address of a variable

&variable returns an **rvalue**



Address-of operator and dereferencing operator

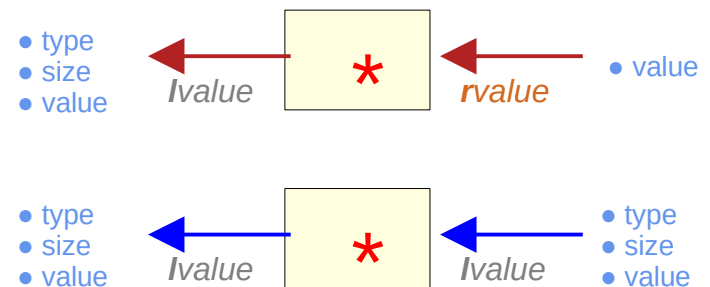
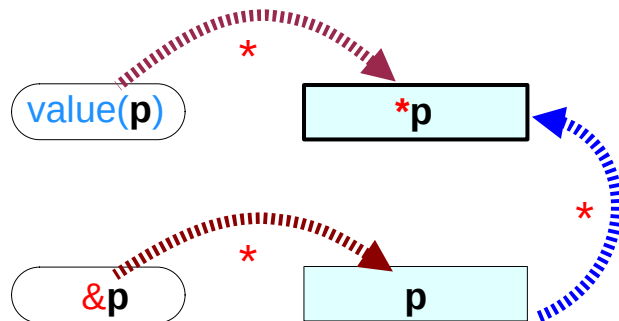
*dereferencing operator ***** : the content at an address*

*an **address** can be an address value or a pointer variable that holds an address value*

*an **address** must be an **rvalue**, or evaluated to be an **rvalue***

****** **address** returns the content value at the address*

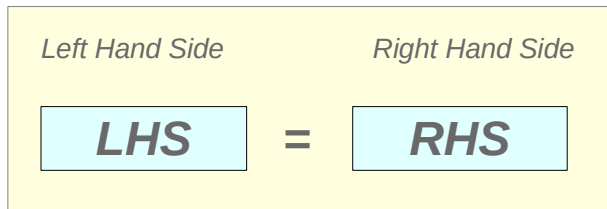
****** **address** returns an **lvalue***



***p** $\equiv$ ***value(p)**

Ivalue and rvalue in assignments

an assignment statement



- in the **LHS**, only **Ivalue** can exist
- **rvalue** can exist only in the **RHS**

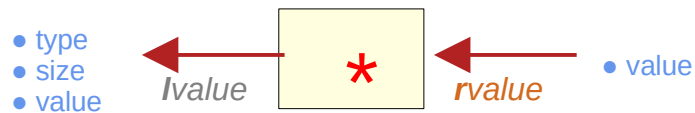
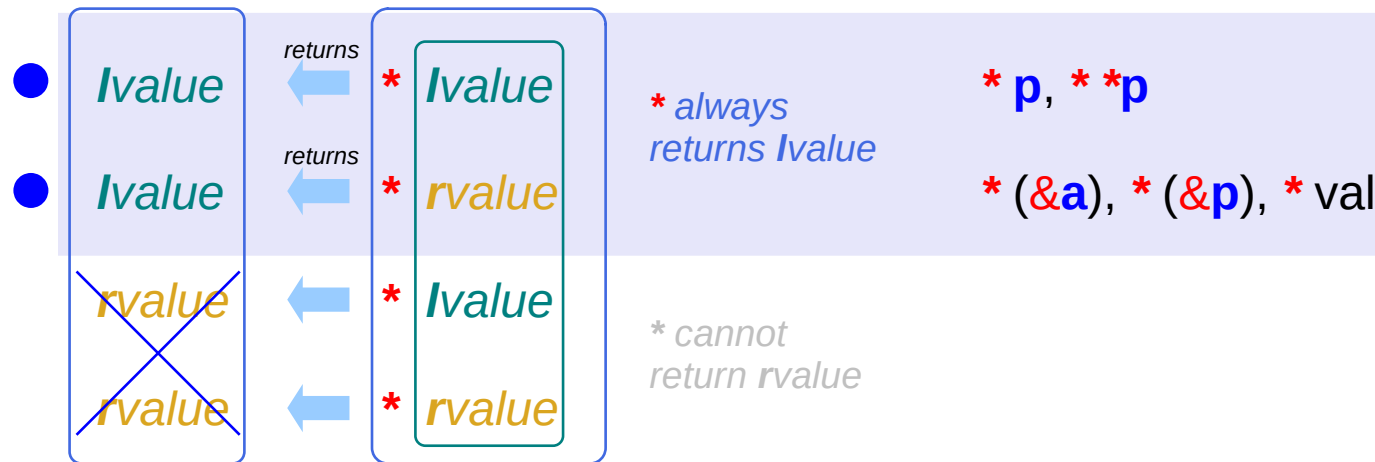
```
int  a, b = 10 ;  
int  *p, *q = &a ;
```

a, b, p, q	: Ivalues	... variables	... RW
*p, *q	: Ivalues	... variables	... RW
&a, &b	: rvalues	... constants	... RO

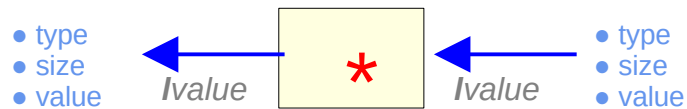
Ivalue	=	Ivalue
Ivalue	=	rvalue
rvalue	=	Ivalue
rvalue	=	rvalue

p	=	q ;
p	=	&a ;
&a	=	p ;
&a	=	&b ;

Ivalue and rvalue with * and & operators



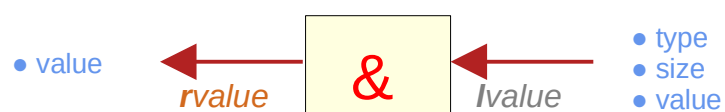
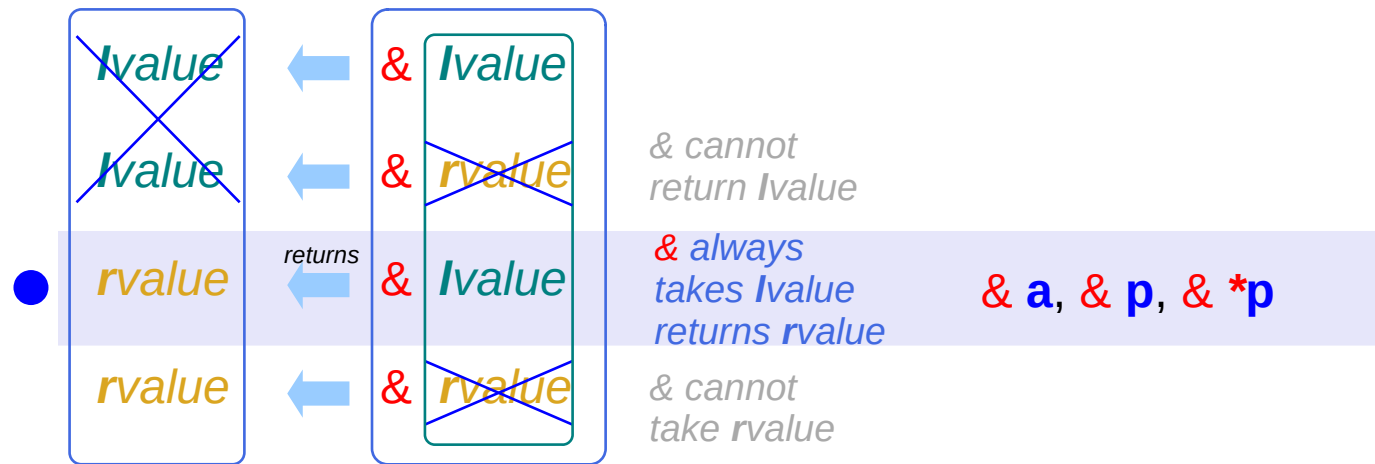
`*` can be applied to either an **lvalue** variable or a **rvalue** address



`*` **operand** becomes an **lvalue** variable thus it can be applied successively.

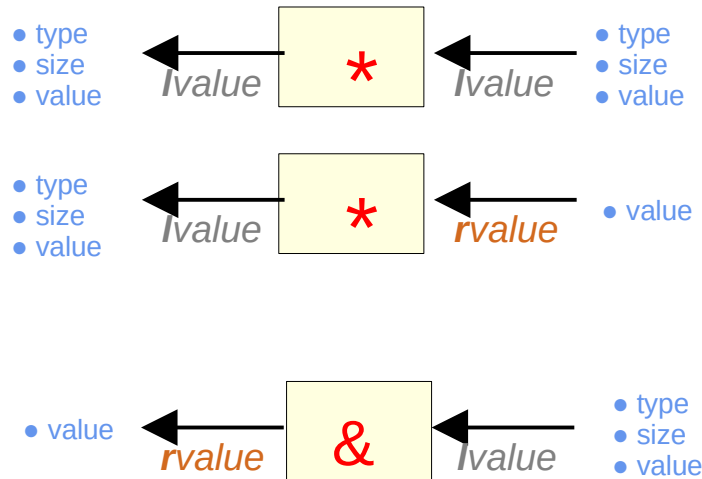
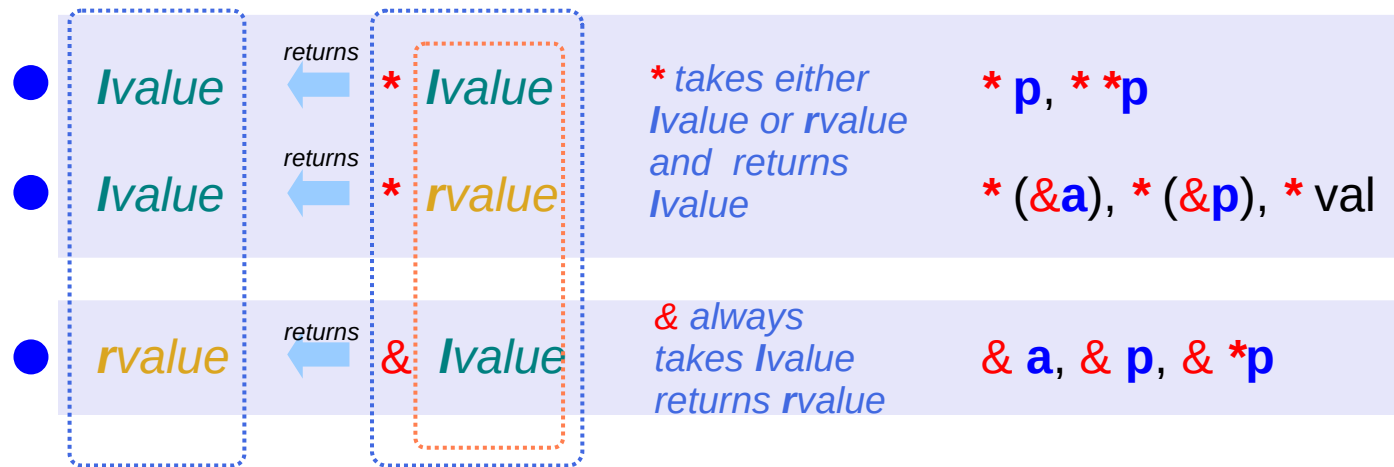
$$*p \equiv *value(p)$$

Ivalue and rvalue with * and & operators



`&` can be applied to only an **lvalue** variable and returns only an **rvalue** address

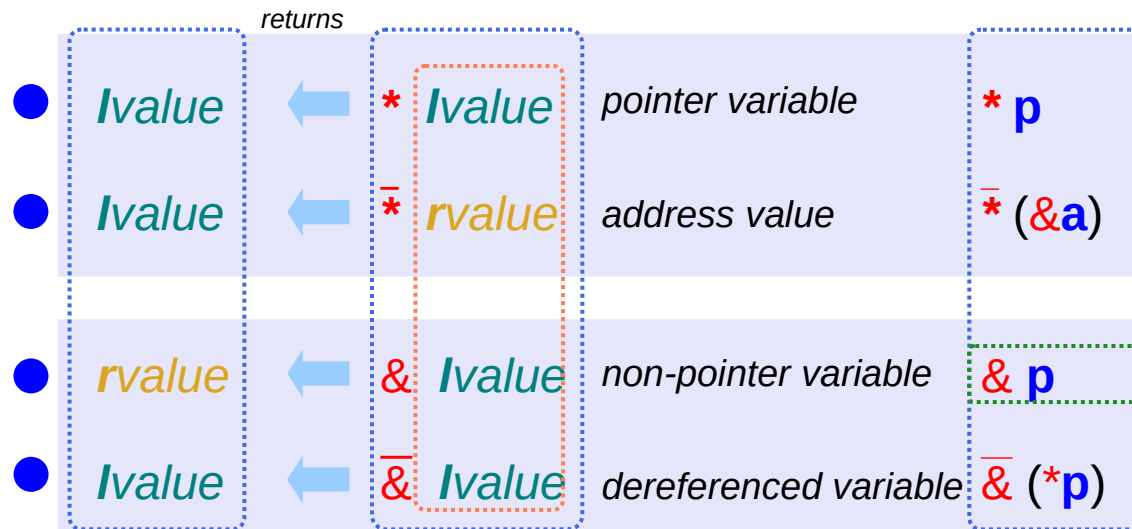
Ivalue and rvalue with * and & operators



* (an **Ivalue** variable)
 * (an **rvalue** address)
 * **operand** : an **Ivalue** variable
 * can be applied successively.

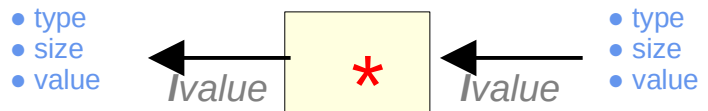
& (an **Ivalue** variable)
 & **operand** : an **rvalue** address,
 & cannot be applied successively.

Ivalue and rvalue with * and & operators

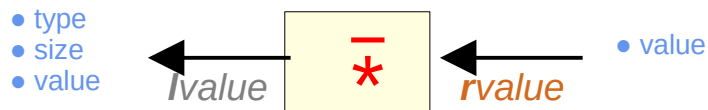


* can be applied successively.
 $\overline{\&}$ can be applied successively.

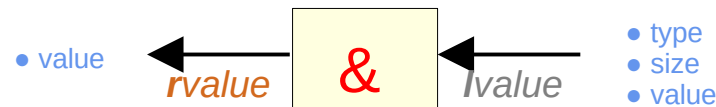
****p**
 $\overline{\&}^{**}p = *p$
 $\overline{\&\&}^{**}p = \overline{\&}^{*}p = p$



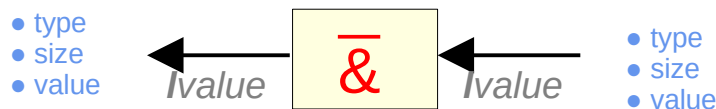
* (an **Ivalue** variable)
 * **operand**: an **Ivalue** variable



$\overline{*}$ (an **rvalue** address)
 $\overline{*}$ **operand**: an **Ivalue** variable



& (an **Ivalue** variable)
 & **operand**: an **rvalue** address



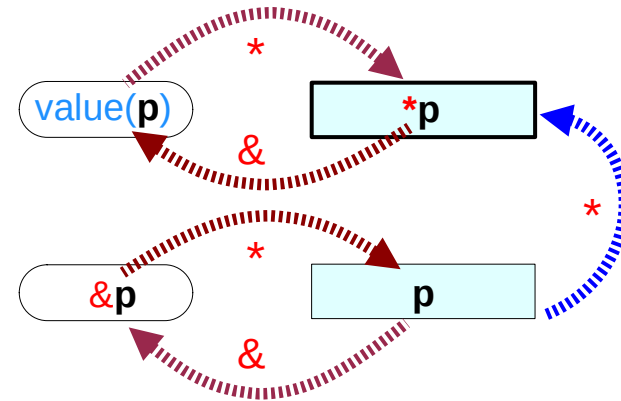
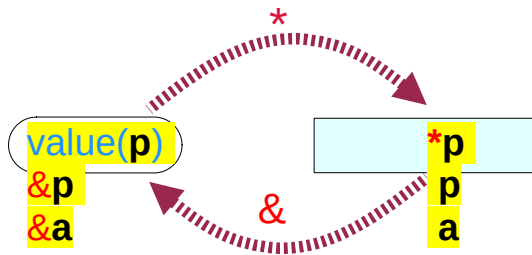
$\overline{\&}$ (an dereferenced **Ivalue** variable)
 $\overline{\&}$ **operand**: an **Ivalue** variable

Variable types

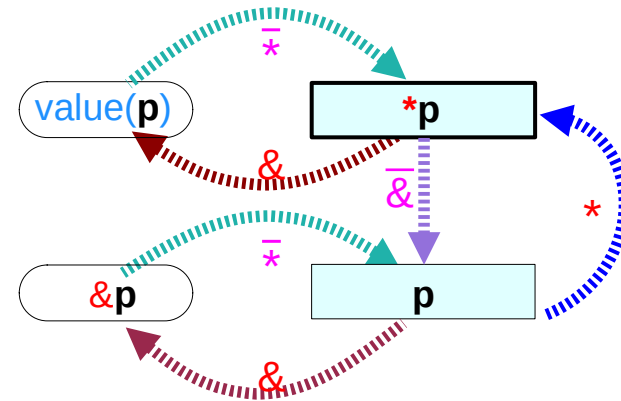
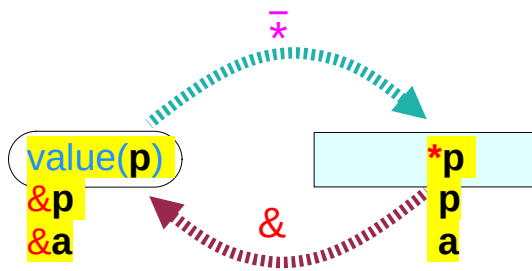
a	non-pointer variables	<i>lvalue</i>
p	pointer variables	<i>lvalue</i>
*p	dereferenced variables	<i>lvalue</i>
val	address values	<i>rvalue</i>

Comparisons (1)

C operators : $*$, $\&$



augmented operators : $\bar{*}$, $\bar{\&}$



Comparisons (2)

C operators : *, &

	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	val	<i>rvalue</i>

<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>

augmented operators : $\bar{*}$, $\bar{\&}$

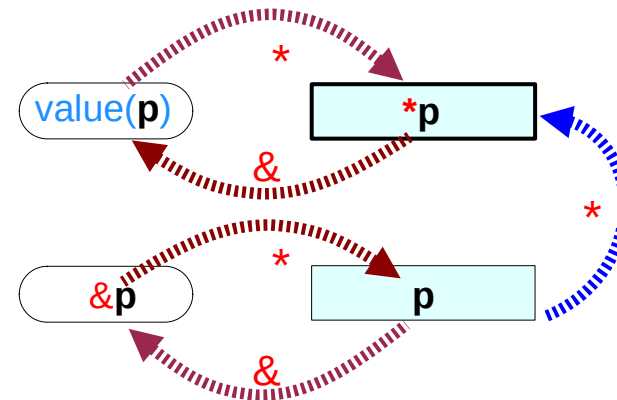
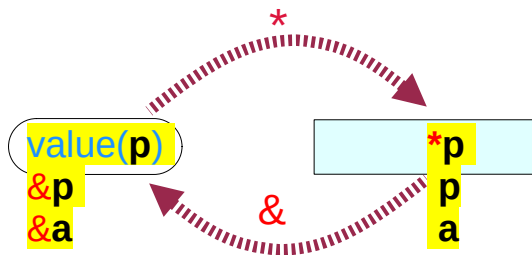
	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{*}$	val	<i>rvalue</i>

<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	$\bar{\&}$	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>

C operators : *, &

	×	a	<i>Ivalue</i>
<i>Ivalue</i> ←	*	p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	*p	<i>Ivalue</i>
<i>Ivalue</i> ←	*	val	<i>rvalue</i>

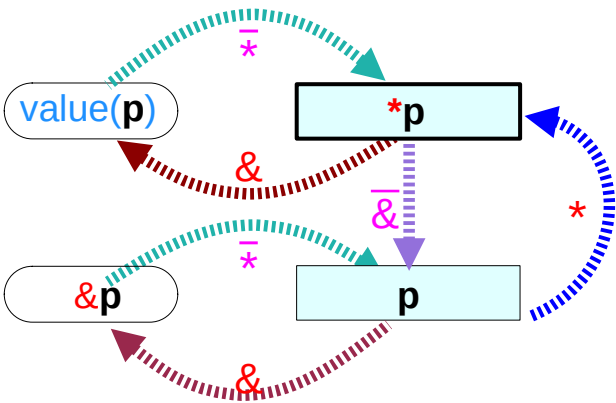
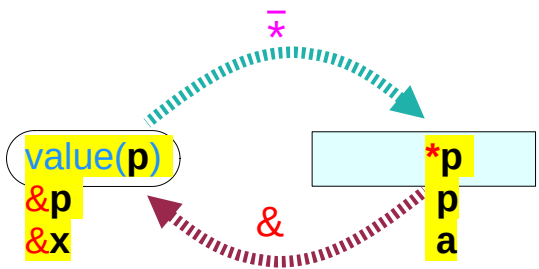
<i>rvalue</i> ←	&	a	<i>Ivalue</i>
<i>rvalue</i> ←	&	p	<i>Ivalue</i>
<i>rvalue</i> ←	&	*p	<i>Ivalue</i>
	×	val	<i>rvalue</i>



Augmented operators : $\bar{*}$, $\bar{\&}$

	$\times$	a	<i>Ivalue</i>
<i>Ivalue</i> $\leftarrow$	$*$	p	<i>Ivalue</i>
<i>Ivalue</i> $\leftarrow$	$*$	*p	<i>Ivalue</i>
<i>Ivalue</i> $\leftarrow$	$\bar{*}$	val	<i>rvalue</i>

<i>rvalue</i> $\leftarrow$	$\&$	a	<i>Ivalue</i>
<i>rvalue</i> $\leftarrow$	$\&$	p	<i>Ivalue</i>
<i>rvalue</i> $\leftarrow$	$\&$	*p	<i>Ivalue</i>
<i>Ivalue</i> $\leftarrow$	$\bar{\&}$	*p	<i>Ivalue</i>
	$\times$	val	<i>rvalue</i>



Recursive application of the address-of operator

~~$\&(\&(\&(c[i])[j])[k])$~~

$\&$ C operator

takes only **lvalue** variable

returns **rvalue** **address value**

thus, the above expression is not possible

successive application of $\&$ is not possible,
Unless intermediate variables and proper
type casting

but, $*p$ becomes a **lvalue** variable

$*$ operator can be applied successively.

$\overline{\&}(\overline{\&}(\overline{\&}(c[i])[j])[k])$

$\overline{\&}$ mathematical operator

takes a dereferenced **lvalue** variable

returns **lvalue** variable

successive application of $\overline{\&}$ is not possible

but, $*p$ becomes a **lvalue** variable

$*$ operator can be applied successively.

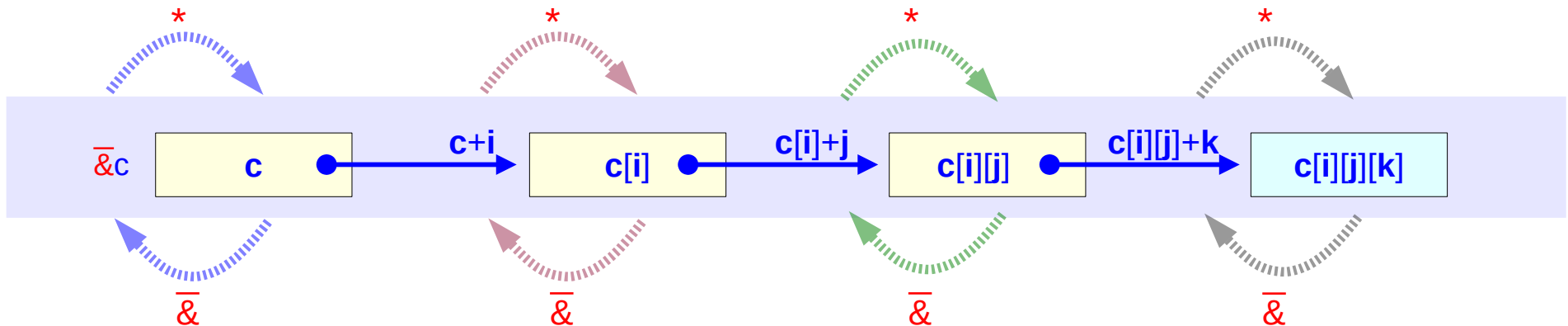
Recursive application of the address-of operator

$$*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$$

$$\begin{aligned} &*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k}) = \\ &*(*(\mathbf{c}[\mathbf{i}]+\mathbf{j})+\mathbf{k}) = \\ &*(\mathbf{c}[\mathbf{i}][\mathbf{j}]+\mathbf{k}) = \\ &\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}] \end{aligned}$$

$$\overline{\&}(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}])$$

$$\begin{aligned} &\overline{\&}(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}]) = \\ &(\overline{\&}(\overline{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])+\mathbf{k}) = \\ &((\overline{\&}(\mathbf{c}[\mathbf{i}])+\mathbf{j})+\mathbf{k}) = \\ &(((\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k}) \end{aligned}$$



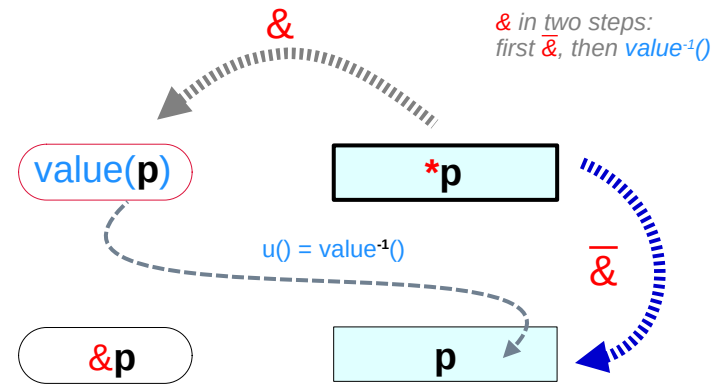
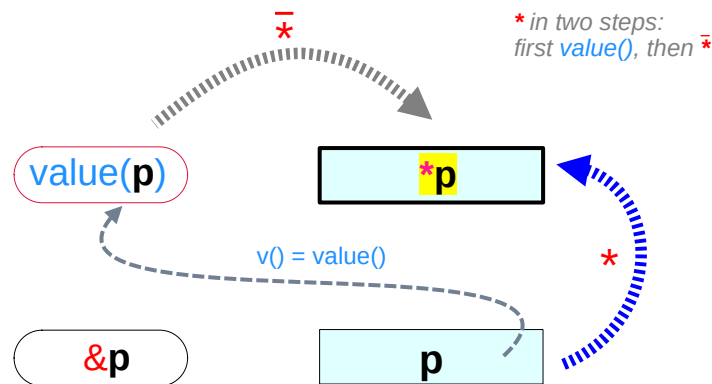
Recursive application of the address-of operator

$\ast(\ast(\ast(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$\bar{\ast}\bar{v}(\bar{\ast}\bar{v}(\bar{\ast}\bar{v}(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$\bar{\&}(\bar{\&}(\bar{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}])$

$\bar{u}\bar{\&}(\bar{u}\bar{\&}(\bar{u}\bar{\&}(\mathbf{c}[\mathbf{i}])[\mathbf{j}])[\mathbf{k}])$



Recursive application of the address-of operator

successive application of `&` is not possible,
Unless intermediate variables and proper
type casting

~~$v \& (v \& (v \& (c[i][j])[k]))$~~

~~$\&(\&(\&(c[i][j])[k]))$~~

$$\begin{aligned} \&(c[i][j][k]) &= \&(* (c[i][j] + k)) &= \&(\bar{*}v(c[i][j] + k)) &= v(c[i][j] + k) \\ & & & &= c[i][j] + k * \text{sizeof}(c[0][0][0]) \end{aligned}$$

$$\begin{aligned} \&(c[i][j]) &= \&(* (c[i] + j)) &= \&(\bar{*}v(c[i] + j)) &= v(c[i] + j) \\ & & & &= c[i] + j * \text{sizeof}(c[0][0]) \end{aligned}$$

$$\begin{aligned} \&(c[i]) &= \&(* (c + i)) &= \&(\bar{*}v(c + i)) &= v(c + i) \\ & & & &= c + i * \text{sizeof}(c[0]) \end{aligned}$$

Recursive application of the address-of operator

$*(*(*(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$\overline{*}\mathbf{v}(\overline{*}\mathbf{v}(\overline{*}\mathbf{v}(\mathbf{c}+\mathbf{i})+\mathbf{j})+\mathbf{k})$

$*(*(\mathbf{c}[\mathbf{i}]+\mathbf{j})+\mathbf{k})$

$*(\mathbf{c}[\mathbf{i}][\mathbf{j}]+\mathbf{k})$

$\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]$

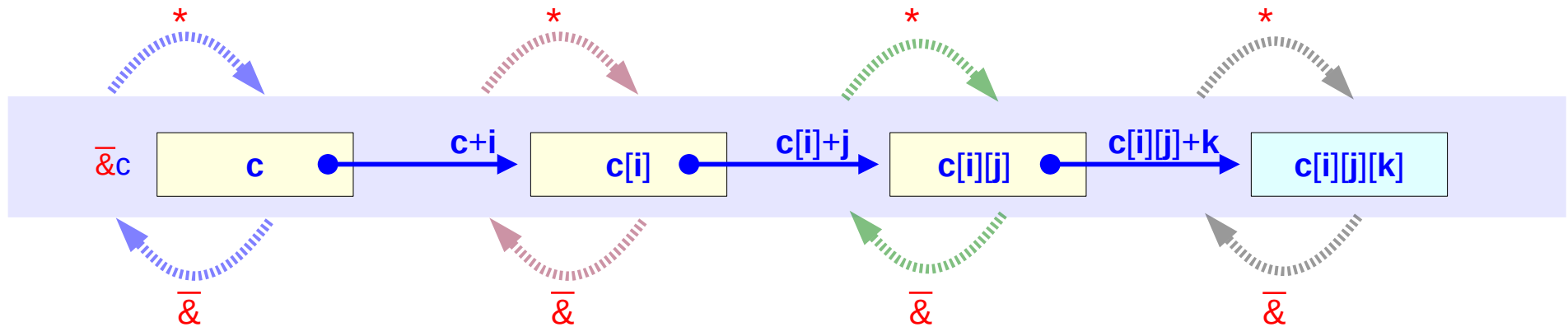
$\overline{\&}(\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}])$	$= \mathbf{c}[\mathbf{i}][\mathbf{j}]$	$+ \mathbf{k}$
$\overline{\&}(\mathbf{c}[\mathbf{i}][\mathbf{j}])$	$= \mathbf{c}[\mathbf{i}]$	$+ \mathbf{j}$
$\overline{\&}(\mathbf{c}[\mathbf{i}])$	$= \mathbf{c}$	$+ \mathbf{i}$

$\mathbf{c}[\mathbf{i}][\mathbf{j}][\mathbf{k}]$	$= *(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})$
$\mathbf{c}[\mathbf{i}][\mathbf{j}]$	$= *(\mathbf{c}[\mathbf{i}] + \mathbf{j})$
$\mathbf{c}[\mathbf{i}]$	$= *(\mathbf{c} + \mathbf{i})$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}
 \overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\
 \overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\
 \overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i
 \end{aligned}$$

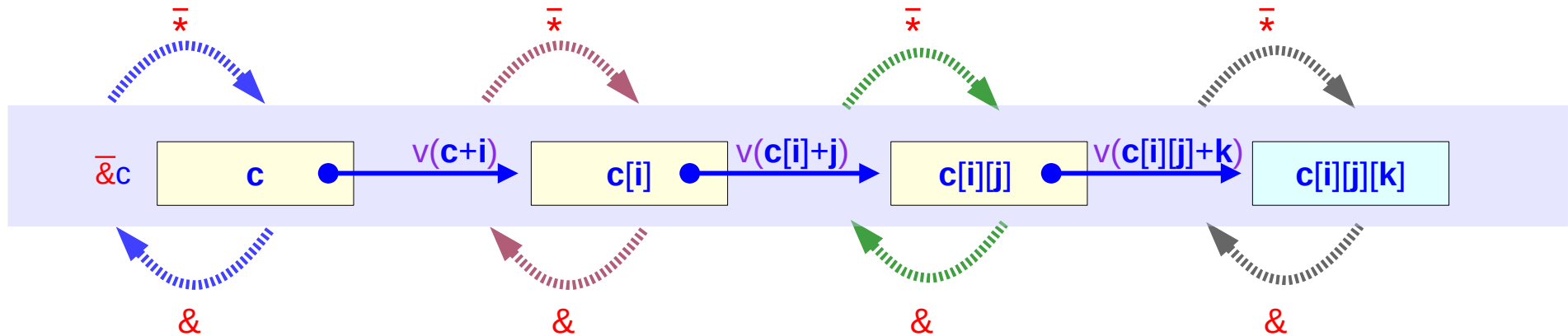
$$\begin{aligned}
 * \overline{\&}(c[i][j][k]) &= * \overline{\&}(* (c[i][j] + k)) &= *(c[i][j] + k) \\
 * \overline{\&}(c[i][j]) &= * \overline{\&}(* (c[i] + j)) &= *(c[i] + j) \\
 * \overline{\&}(c[i]) &= * \overline{\&}(* (c + i)) &= *(c + i)
 \end{aligned}$$



Two step deferencing in type II (1) – without skipping

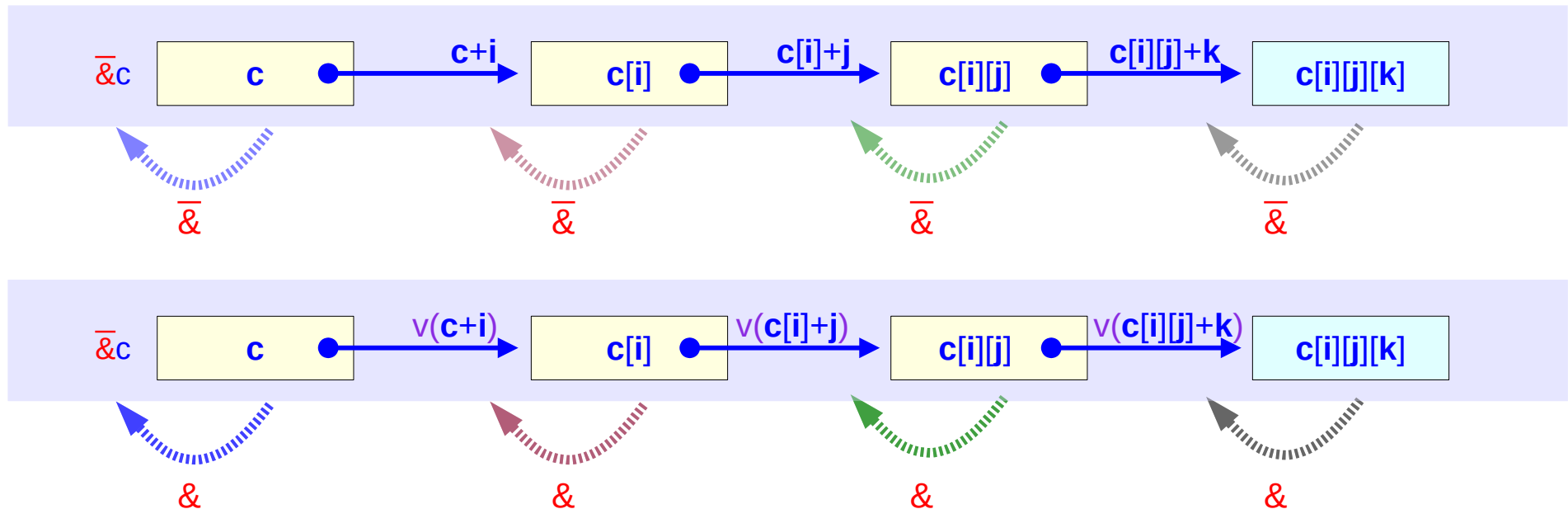
$$\begin{aligned}
 \&(\mathbf{c[i][j][k]}) &= \&(*(\mathbf{c[i][j]}+\mathbf{k})) &= \text{value}(\mathbf{c[i][j]} + \mathbf{k}) &= \mathbf{c[i][j]} + \mathbf{k} * \text{sizeof}(\mathbf{c[0][0][0]}) \\
 \&(\mathbf{c[i][j]}) &= \&(*(\mathbf{c[i]}+\mathbf{j})) &= \text{value}(\mathbf{c[i]} + \mathbf{j}) &= \mathbf{c[i]} + \mathbf{j} * \text{sizeof}(\mathbf{c[0][0]}) \\
 \&(\mathbf{c[i]}) &= \&(*(\mathbf{c}+\mathbf{i})) &= \text{value}(\mathbf{c} + \mathbf{i}) &= \mathbf{c} + \mathbf{i} * \text{sizeof}(\mathbf{c[0]})
 \end{aligned}$$

$$\begin{aligned}
 \bar{*}\&(\mathbf{c[i][j][k]}) &= \bar{*}\&(*(\mathbf{c[i][j]}+\mathbf{k})) &= \bar{*}\text{value}(\mathbf{c[i][j]} + \mathbf{k}) &= \bar{*}(\mathbf{c[i][j]} + \mathbf{k} * \text{sizeof}(\mathbf{c[0][0][0]})) \\
 \bar{*}\&(\mathbf{c[i][j]}) &= \bar{*}\&(*(\mathbf{c[i]}+\mathbf{j})) &= \bar{*}\text{value}(\mathbf{c[i]} + \mathbf{j}) &= \bar{*}(\mathbf{c[i]} + \mathbf{j} * \text{sizeof}(\mathbf{c[0][0]})) \\
 \bar{*}\&(\mathbf{c[i]}) &= \bar{*}\&(*(\mathbf{c}+\mathbf{i})) &= \bar{*}\text{value}(\mathbf{c} + \mathbf{i}) &= \bar{*}(\mathbf{c} + \mathbf{i} * \text{sizeof}(\mathbf{c[0]}))
 \end{aligned}$$



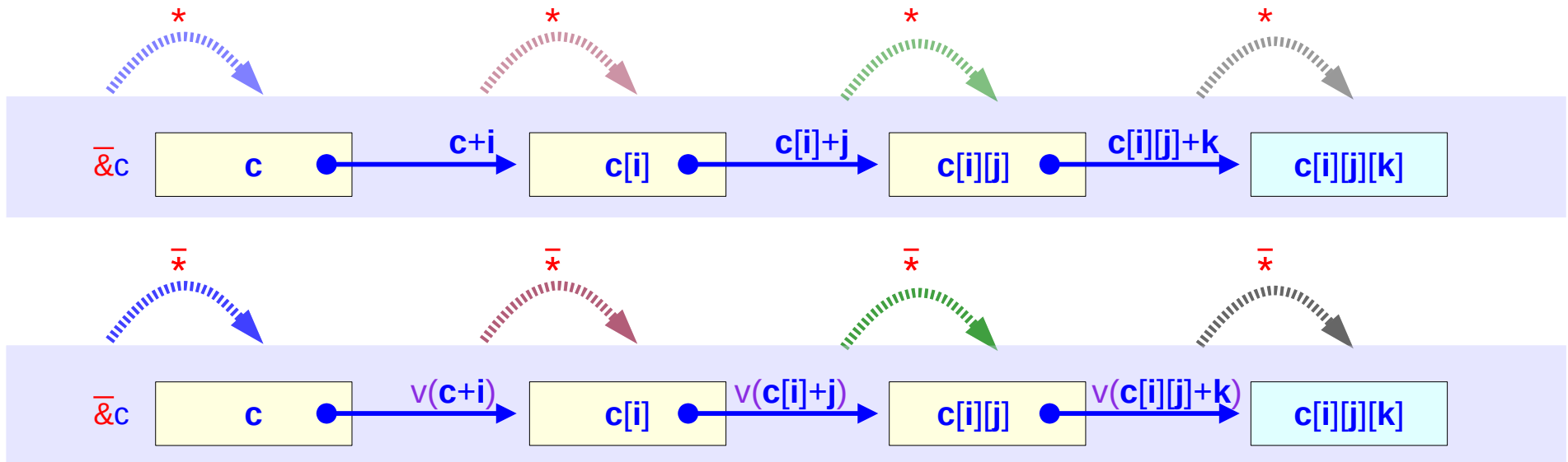
$$\begin{aligned}
 \overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\
 \overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\
 \overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i
 \end{aligned}$$

$$\begin{aligned}
 \&(c[i][j][k]) &= \&(* (c[i][j] + k)) &= c[i][j] &+ k * \text{sizeof}(c[0][0][0]) \\
 \&(c[i][j]) &= \&(* (c[i] + j)) &= c[i] &+ j * \text{sizeof}(c[0][0]) \\
 \&(c[i]) &= \&(* (c + i)) &= c &+ i * \text{sizeof}(c[0])
 \end{aligned}$$



$$\begin{aligned}
*\bar{\&}(c[i][j][k]) &= *\bar{\&}(* (c[i][j] + k)) &= *(c[i][j] + k) \\
*\bar{\&}(c[i][j]) &= *\bar{\&}(* (c[i] + j)) &= *(c[i] + j) \\
*\bar{\&}(c[i]) &= *\bar{\&}(* (c + i)) &= *(c + i)
\end{aligned}$$

$$\begin{aligned}
*\bar{\&}(c[i][j][k]) &= *\bar{\&}(* (c[i][j] + k)) &= *\bar{\&}(c[i][j] + k * \text{sizeof}(c[0][0][0])) \\
*\bar{\&}(c[i][j]) &= *\bar{\&}(* (c[i] + j)) &= *\bar{\&}(c[i] + j * \text{sizeof}(c[0][0])) \\
*\bar{\&}(c[i]) &= *\bar{\&}(* (c + i)) &= *\bar{\&}(c + i * \text{sizeof}(c[0]))
\end{aligned}$$



Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= \overline{\&}(* (c[i][j] + k)) &= c[i][j] &+ k \\ \overline{\&}(c[i][j]) &= \overline{\&}(* (c[i] + j)) &= c[i] &+ j \\ \overline{\&}(c[i]) &= \overline{\&}(* (c + i)) &= c &+ i\end{aligned}$$

$$\begin{aligned}* \overline{\&}(c[i][j][k]) &= * \overline{\&}(* (c[i][j] + k)) &= *(c[i][j] + k) \\ * \overline{\&}(c[i][j]) &= * \overline{\&}(* (c[i] + j)) &= *(c[i] + j) \\ * \overline{\&}(c[i]) &= * \overline{\&}(* (c + i)) &= *(c + i)\end{aligned}$$

$$\begin{aligned}\&(c[i][j][k]) &= \&(* (c[i][j] + k)) &= c[i][j] &+ k * \text{sizeof}(c[0][0][0]) \\ \&(c[i][j]) &= \&(* (c[i] + j)) &= c[i] &+ j * \text{sizeof}(c[0][0]) \\ \&(c[i]) &= \&(* (c + i)) &= c &+ i * \text{sizeof}(c[0])\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= * \overline{\&}(* (c[i][j] + k)) &= \overline{*}(c[i][j] + k * \text{sizeof}(c[0][0][0])) \\ c[i][j] &= * \overline{\&}(* (c[i] + j)) &= \overline{*}(c[i] + j * \text{sizeof}(c[0][0])) \\ c[i] &= * \overline{\&}(* (c + i)) &= \overline{*}(c + i * \text{sizeof}(c[0]))\end{aligned}$$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= c[i][j] + k \\ \overline{\&}(c[i][j]) &= c[i] + j \\ \overline{\&}(c[i]) &= c + i\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= *(c[i][j] + k) \\ c[i][j] &= *(c[i] + j) \\ c[i] &= *(c + i)\end{aligned}$$

$$\begin{aligned}\&(c[i][j][k]) &= c[i][j] + k * \text{sizeof}(c[0][0][0]) \\ \&(c[i][j]) &= c[i] + j * \text{sizeof}(c[0][0]) \\ \&(c[i]) &= c + i * \text{sizeof}(c[0])\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= \overline{*}(c[i][j] + k * \text{sizeof}(c[0][0][0])) \\ c[i][j] &= \overline{*}(c[i] + j * \text{sizeof}(c[0][0])) \\ c[i] &= \overline{*}(c + i * \text{sizeof}(c[0]))\end{aligned}$$

Two step deferencing in type II (1) – without skipping

$$\begin{aligned}\overline{\&}(c[i][j][k]) &= c[i][j] + k \\ \overline{\&}(c[i][j]) &= c[i] + j \\ \overline{\&}(c[i]) &= c + i\end{aligned}$$

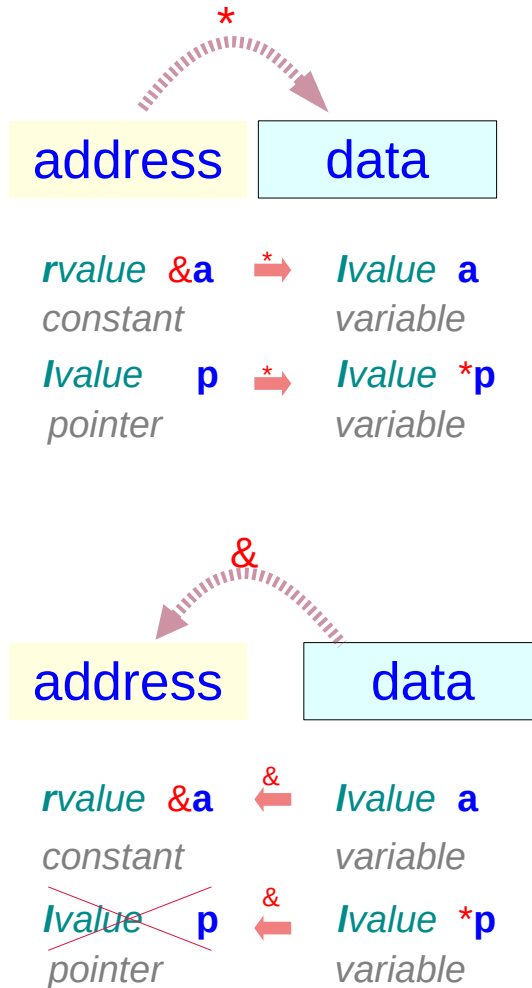
$$\begin{aligned}\&(c[i][j][k]) &= \text{value}(c[i][j] + k) \\ \&(c[i][j]) &= \text{value}(c[i] + j) \\ \&(c[i]) &= \text{value}(c + i)\end{aligned}$$

$$\begin{aligned}c[i][j][k] &= *(c[i][j] + k) \\ c[i][j] &= *(c[i] + j) \\ c[i] &= *(c + i)\end{aligned}$$

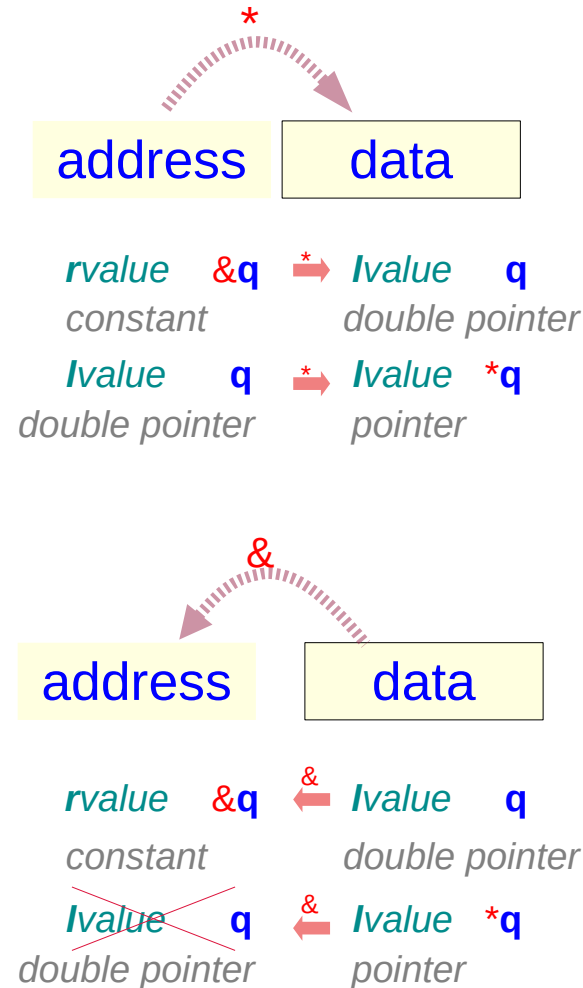
$$\begin{aligned}c[i][j][k] &= \overline{*} \text{value}(c[i][j] + k) \\ c[i][j] &= \overline{*} \text{value}(c[i] + j) \\ c[i] &= \overline{*} \text{value}(c + i)\end{aligned}$$

Address-of & and dereference * C operators (1)

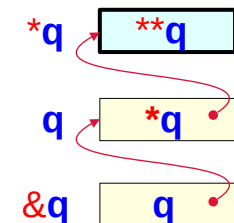
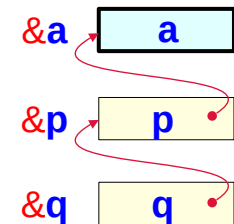
Primitive Data Type



Pointer Data Type

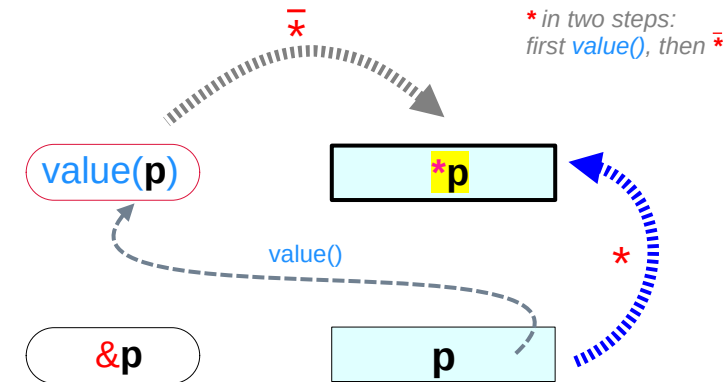
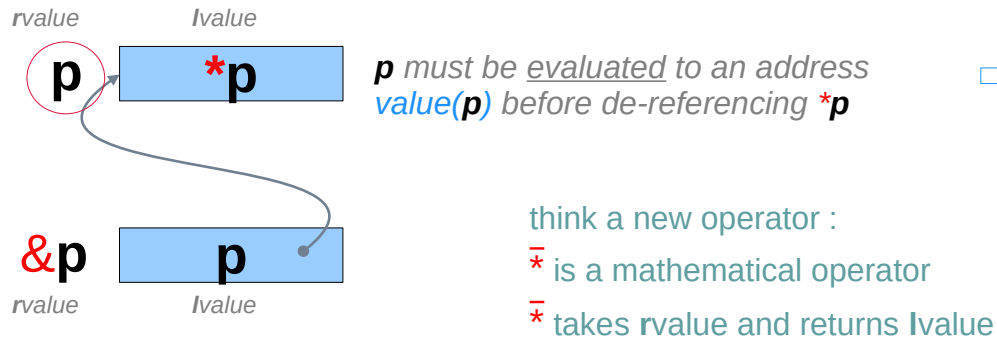


```
int a ;  
int * p ;  
int ** q ;
```

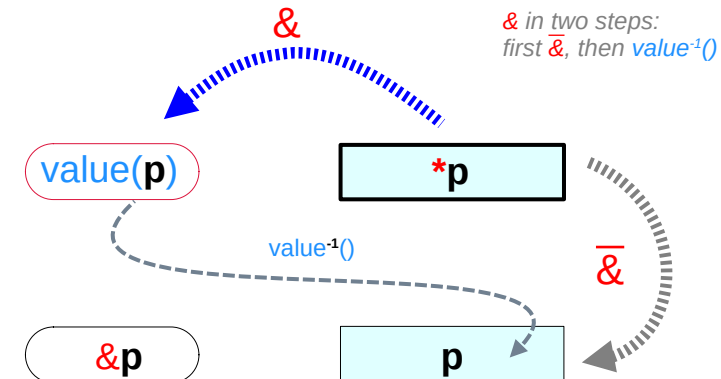
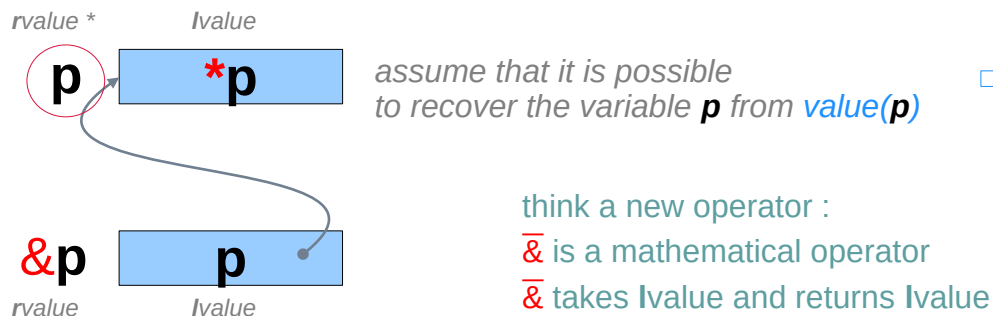


& and * operators in pointer de-referencing

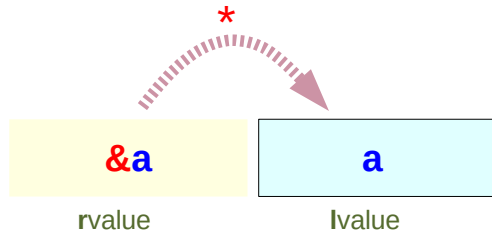
Two step De-reference * operation



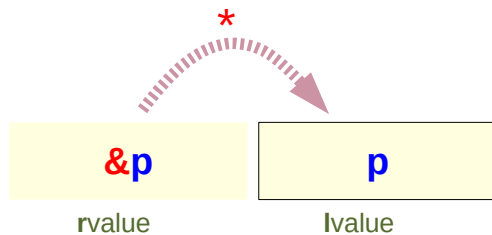
Two step Address-of & operation



Type, Size, and Value attributes of an **lvalue**



lvalue ← **rvalue*
a ***&a**



lvalue ← **rvalue*
p ***&p**

lvalue is associated with a memory location

lvalue has the following attributes

- Type
- Size
- Value

rvalue has the only attribute

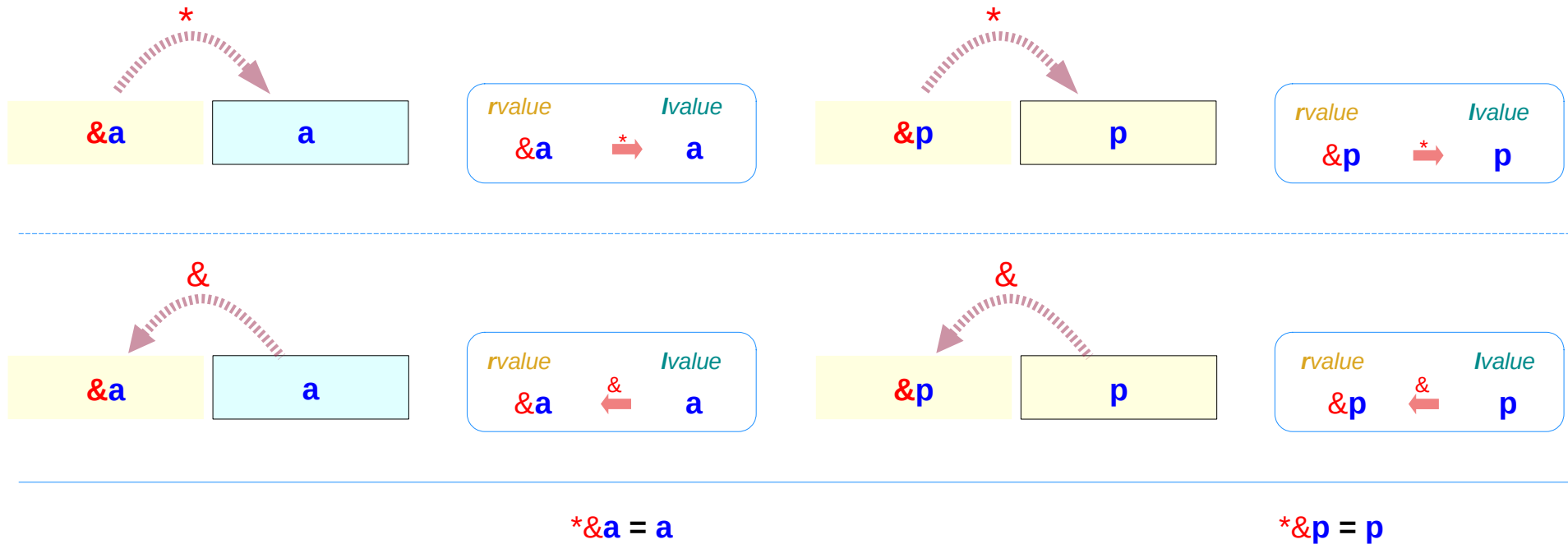
- Value

assume the function `value()`

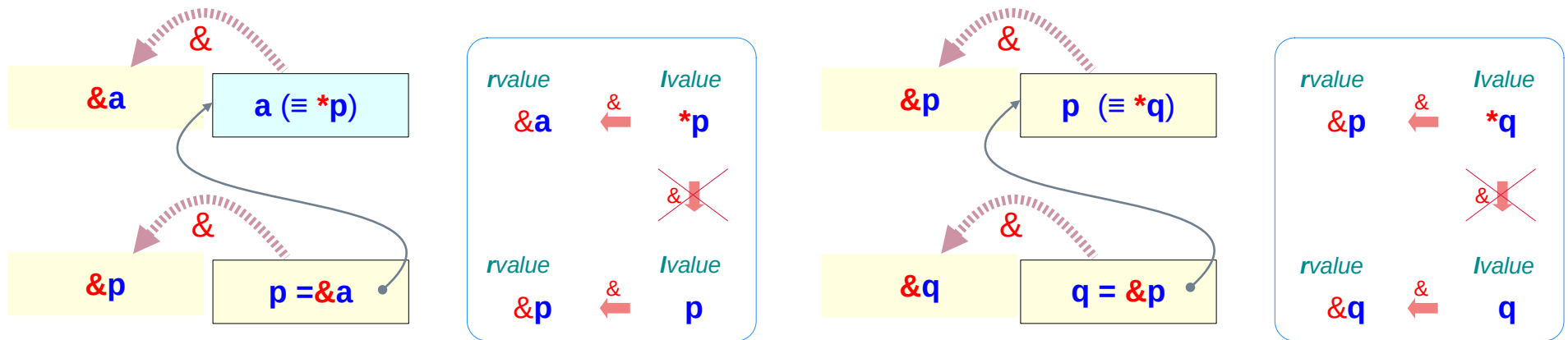
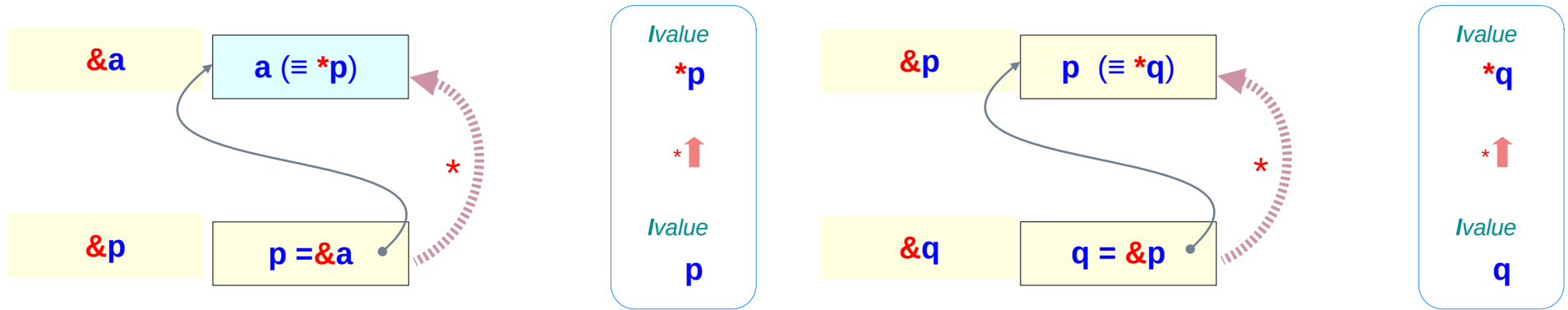
`value(lvalue)` returns
the **Value** attribute of **lvalue**

`value(rvalue)` returns
the **Value** attribute of **rvalue**

Address-of & and dereference * C operators (1)



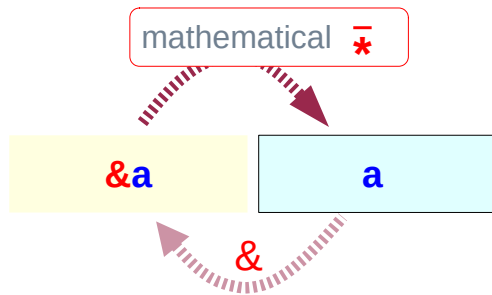
Address-of & and dereference * C operators (2)



$*\&p = p$
 ~~$\&*p = p$~~ *value(p)*

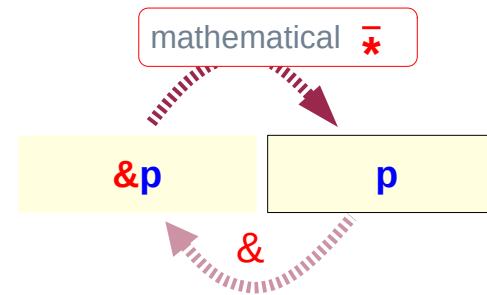
$*\&q = q$
 ~~$\&*q = q$~~ *value(q)*

Introducing mathematical operators : $\bar{\&}$ and $\bar{*}$



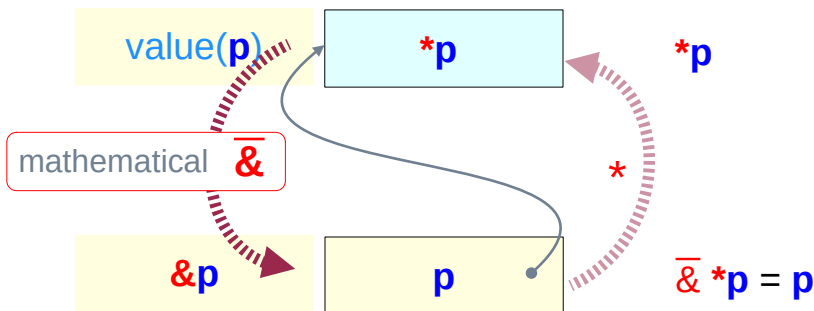
$$\bar{*} \bar{\&a} = a$$

$\bar{\&a}$

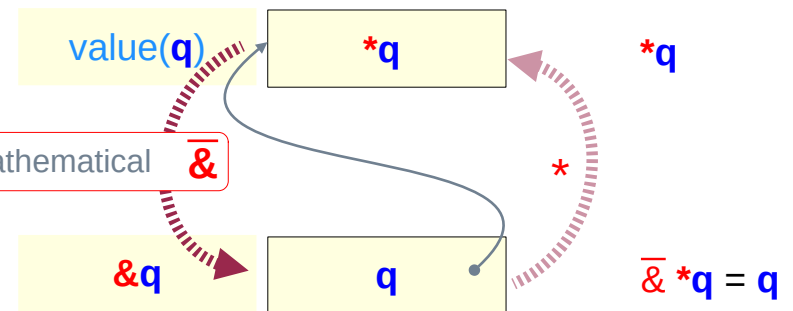


$$\bar{*} \bar{\&p} = p$$

$\bar{\&p}$

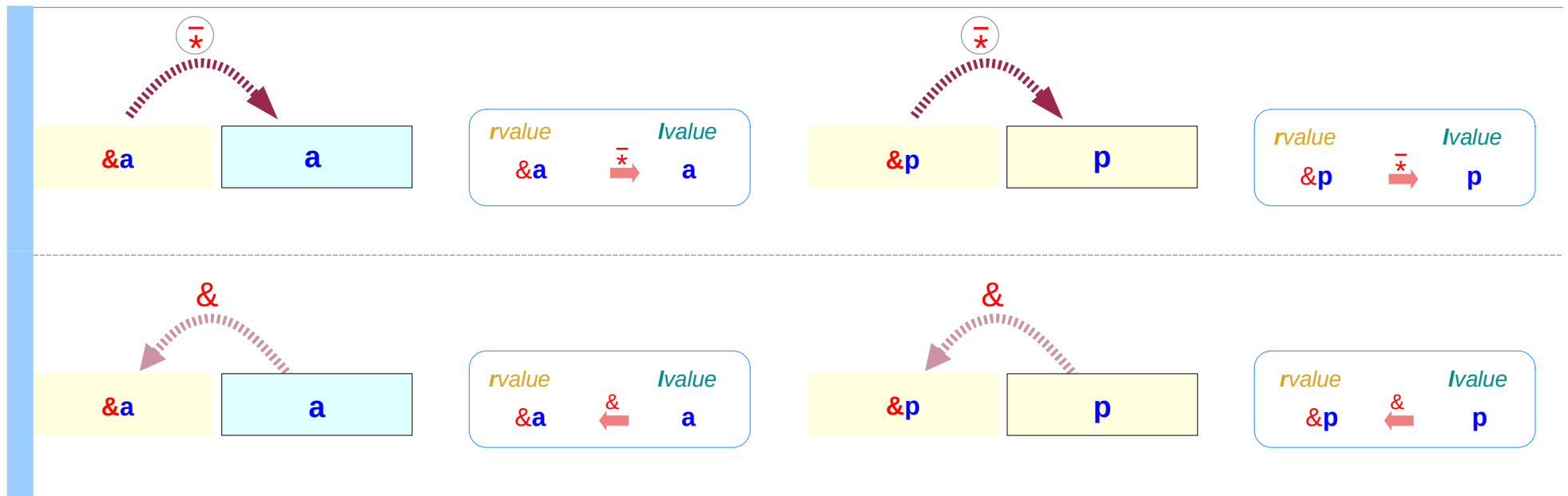


$$\bar{\&} \bar{*}p = p$$



$$\bar{\&} \bar{*}q = q$$

Inverse operators $\bar{*}$ and $\&$



$$\bar{*}\&a = a$$

$$\&\bar{*}\&a = \&a$$

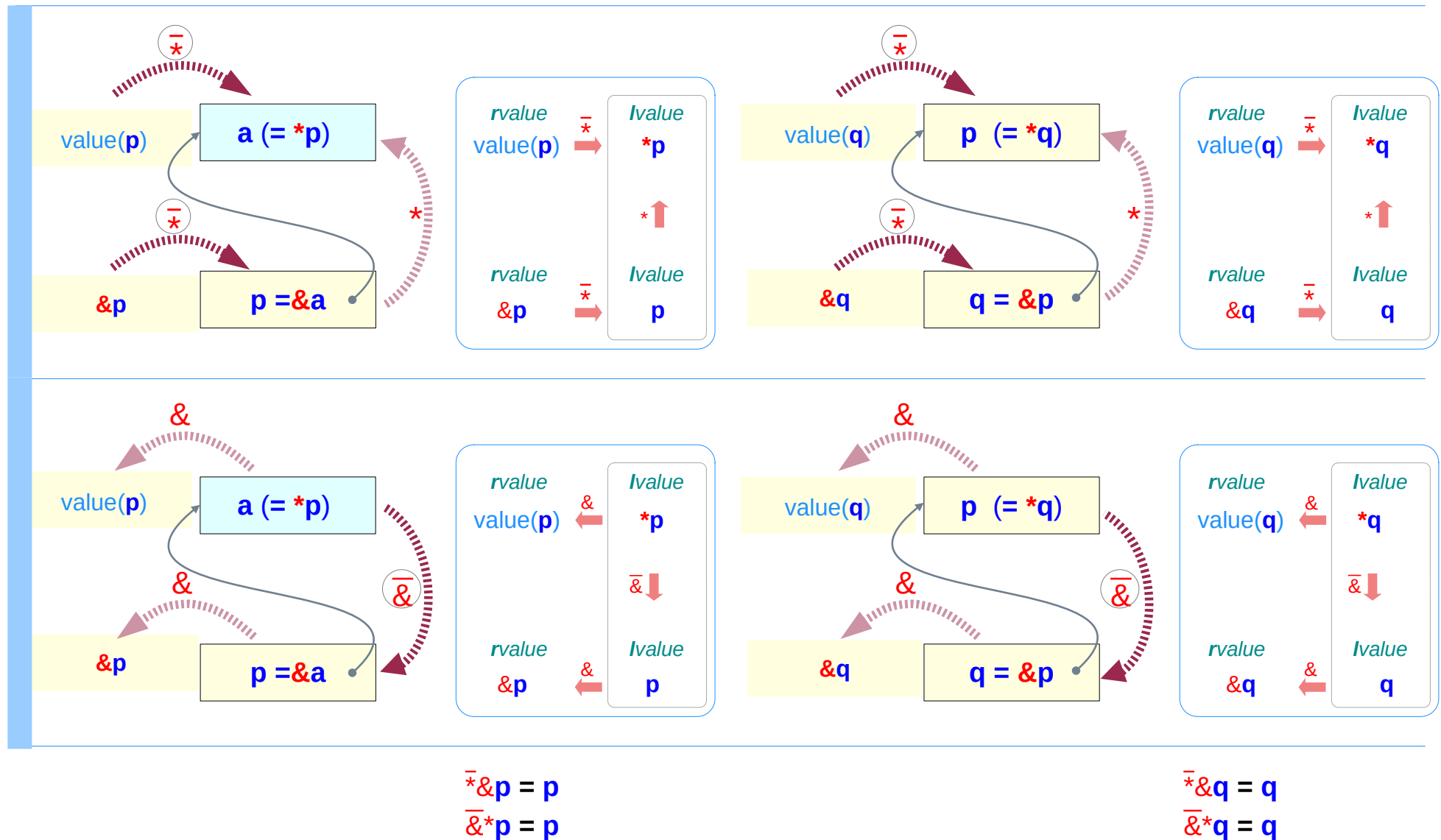
$\bar{*}$ and $\&$ are
inverse operators
to each other

$$\bar{*}\&p = p$$

$$\&\bar{*}\&p = \&p$$

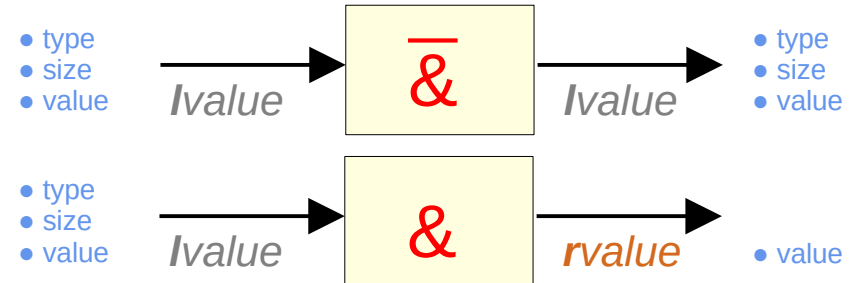
$\bar{*}$ and $\&$ are
inverse operators
to each other

Inverse operators $\bar{*}$ and $*$

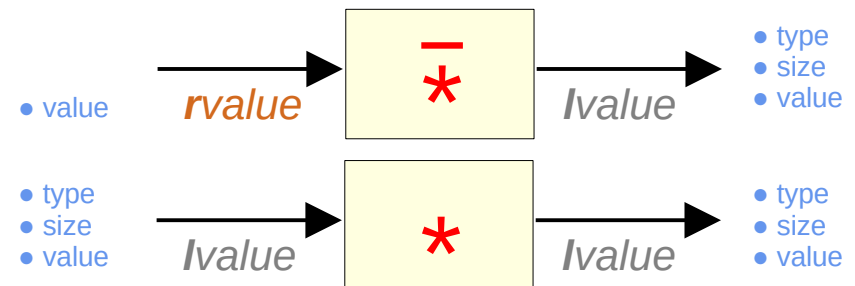
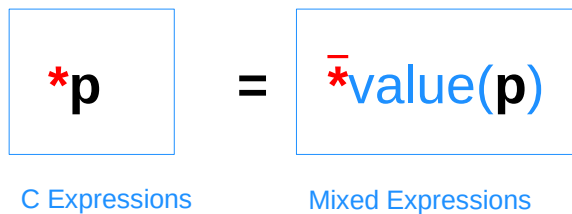


C operators and mathematical operators

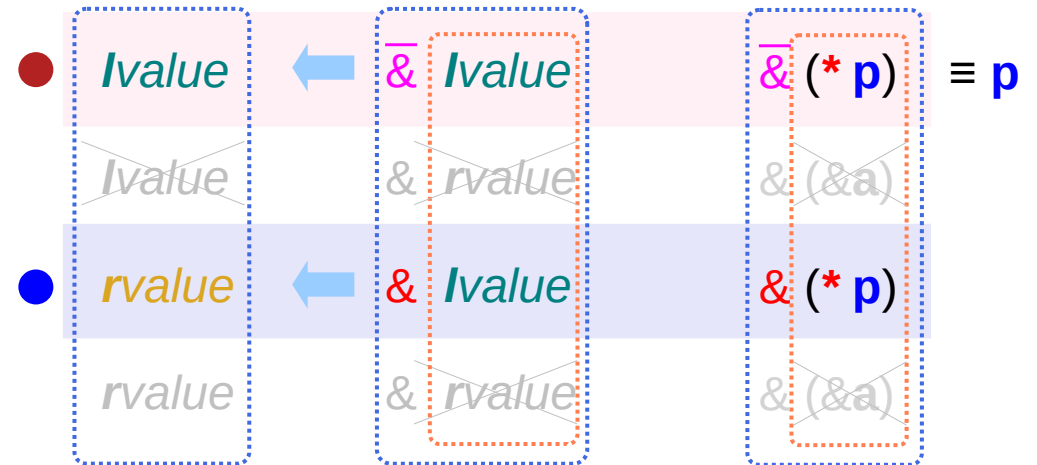
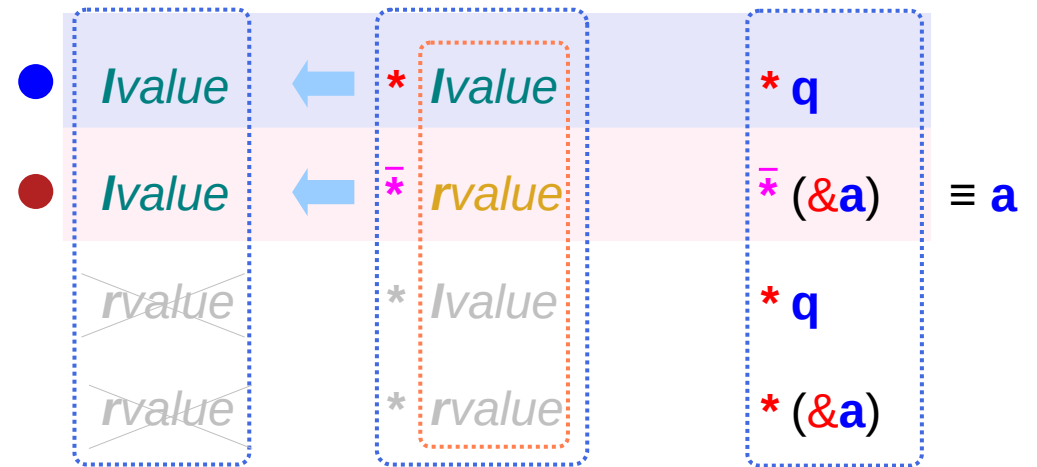
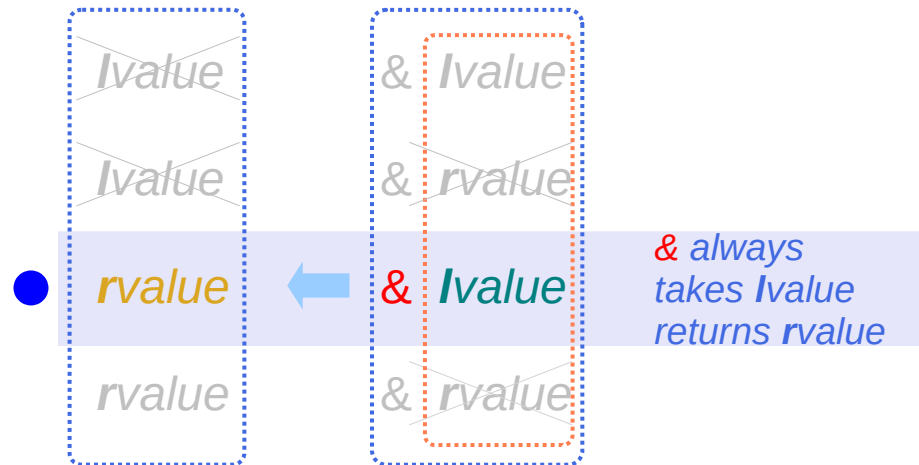
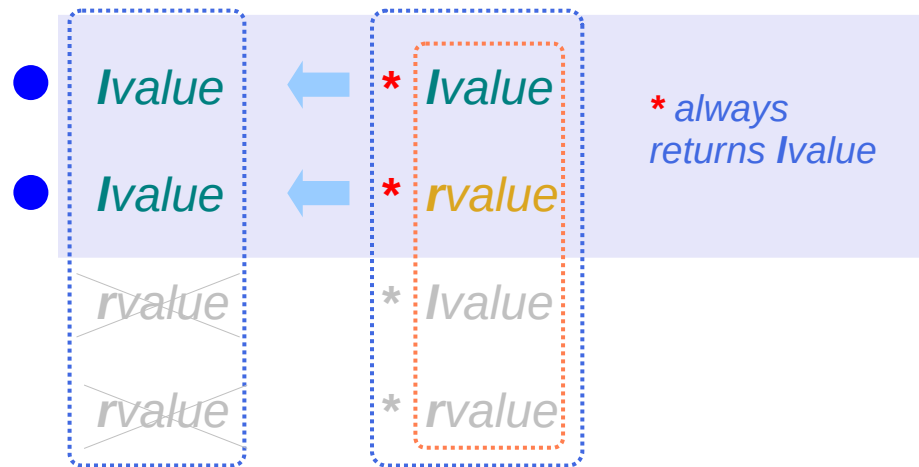
Address-of operation



De-reference operation



Ivalue and rvalue with * and & operators



a, p, q : Ivalues ... variables ... RW
 *p, *q, **q : Ivalues ... variables ... RW
 &a, &p, &q : rvalues ... constants ... RO

Examples of inverse operators

$\overline{*}\&a \rightarrow a$

$\overline{*}\&p \rightarrow p$

$\overline{*}\&q \rightarrow q$

$\overline{*}\text{value}(p) \rightarrow *p \rightarrow a$

$\overline{*}\text{value}(q) \rightarrow *q \rightarrow p$

$\overline{*}\text{value}(*q) \rightarrow **q \rightarrow *p \rightarrow a$

$\overline{\&}*p \rightarrow p$

$\overline{\&}*q \rightarrow q$

$\overline{\&}**q \rightarrow *q$

Extended Operators

$*\&a \rightarrow a$

$*\&p \rightarrow p$

$*\&q \rightarrow q$

$*p \rightarrow a$

$*q \rightarrow p$

$**q \rightarrow *p \rightarrow a$

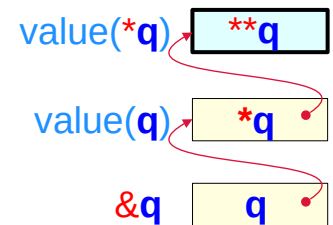
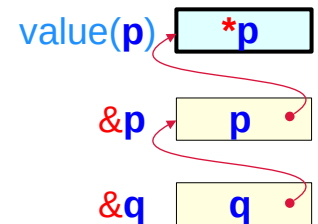
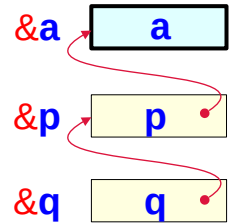
$\&*p \rightarrow \&a$

$\&*q \rightarrow \&p$

$\&**q \rightarrow \&a$

C Operators

```
int a;  
int * p = &a;  
int ** q = &p;
```



Operands of mathematical operators : $\bar{\&}$ and $\bar{*}$

$\bar{*}$ address value

$\bar{\&} \bar{*} \text{value}(\mathbf{P}) \rightarrow \text{value}(\mathbf{P})$

$\bar{*} \bar{\&} \text{variable}$

$\bar{*} \bar{\&} \mathbf{X} \rightarrow \mathbf{X}$

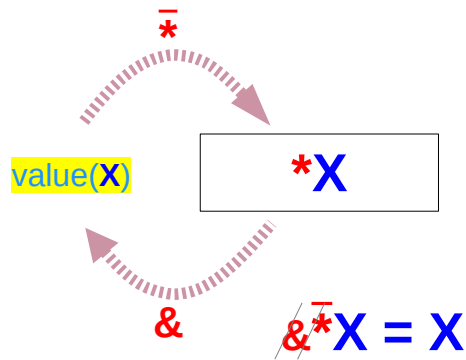
$\bar{\&} \bar{*} \text{pointer}$
dereferenced pointer

$\bar{\&} \bar{*} \mathbf{P} \rightarrow \mathbf{P}$

$\bar{\&} \text{pointer}$
must be a dereferenced pointer
not considering
simultaneous pointing

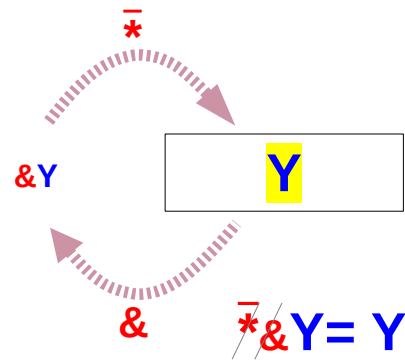
$\bar{*} \bar{\&} \mathbf{Q} \rightarrow \mathbf{Q}$
Q must be a dereferenced pointer
i.e, $\mathbf{Q} = *p \rightarrow \bar{\&} \mathbf{Q} = p$

& and mathematical $\bar{*}$



$\& \bar{*}\text{value}(a) = \text{value}(a)$
 $\& \bar{*}\text{value}(p) = \text{value}(p)$
 $\& \bar{*}\text{value}(q) = \text{value}(q)$

$\& *a = \text{value}(a)$
 $\& *p = \text{value}(p)$
 $\& *q = \text{value}(q)$



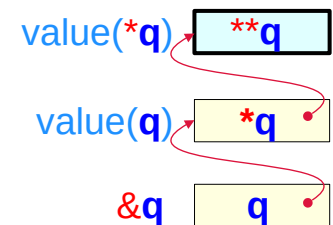
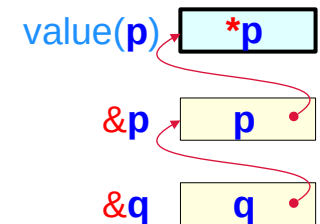
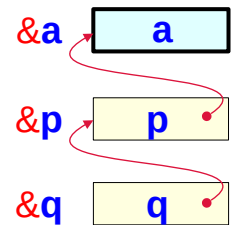
$\bar{*} \&a = a$
 $\bar{*} \&p = p$
 $\bar{*} \&q = q$

$\bar{*} \&a = a$
 $\bar{*} \&p = p$
 $\bar{*} \&q = q$

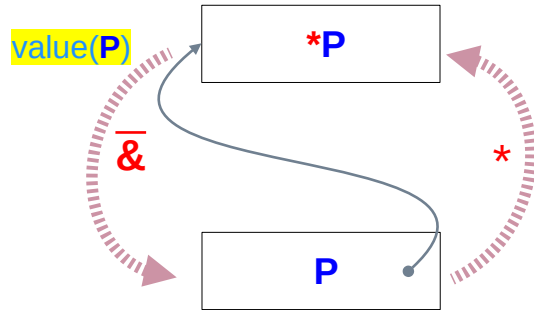
$* \&a = a$
 $* \&p = p$
 $* \&q = q$

C expressions

```
int a;  
int * p = &a;  
int ** q = &p;
```



* and mathematical $\bar{\&}$



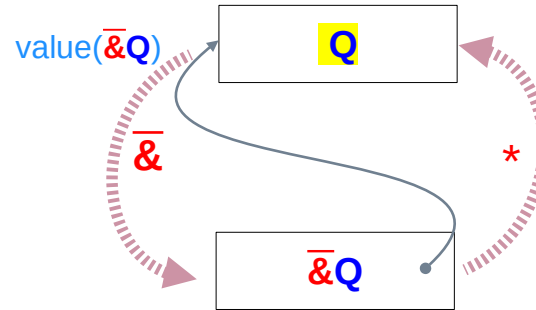
$$\bar{\&*P} = P$$

$$\bar{\&}*p = p$$

$$\bar{\&}*q = q$$

$$\&*p = \text{value}(p)$$

$$\&*q = \text{value}(q)$$



$$*\bar{\&Q} = Q$$

$$*\bar{\&}*p = *p$$

$$*\bar{\&}*q = *q$$

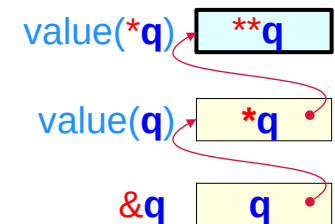
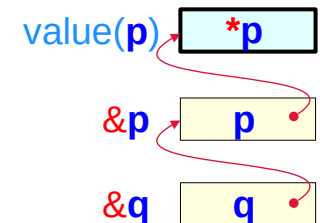
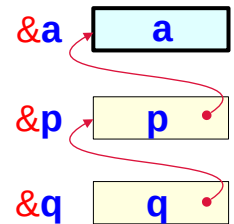
$$\bar{*}\bar{\&}*p = *p$$

$$\bar{*}\bar{\&}*q = *q$$

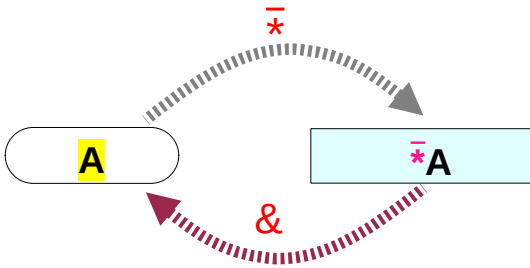
$$\begin{aligned} * \bar{\&}*p &= *p \\ * \bar{\&}*q &= *q \end{aligned}$$

C operators

```
int a;
int * p = &a;
int ** q = &p;
```

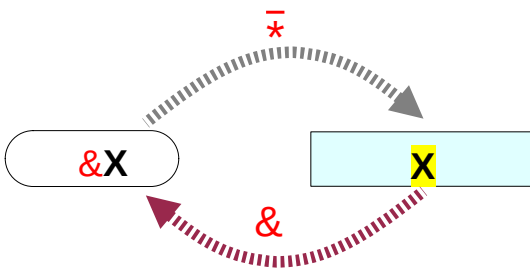


Requirements of address and data

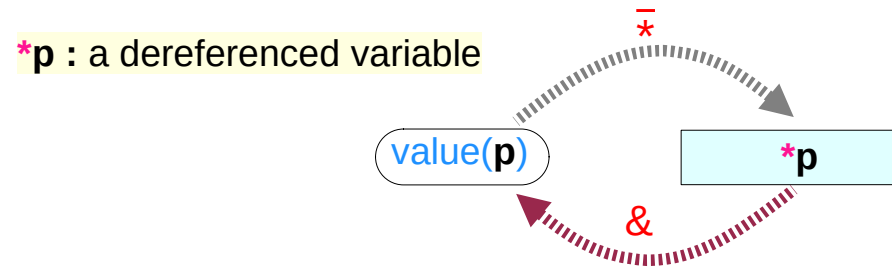


A : an address value - a number

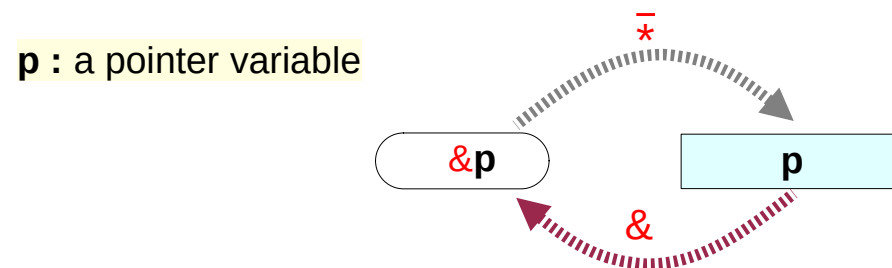
`value(p)`, `&p`, `&x`



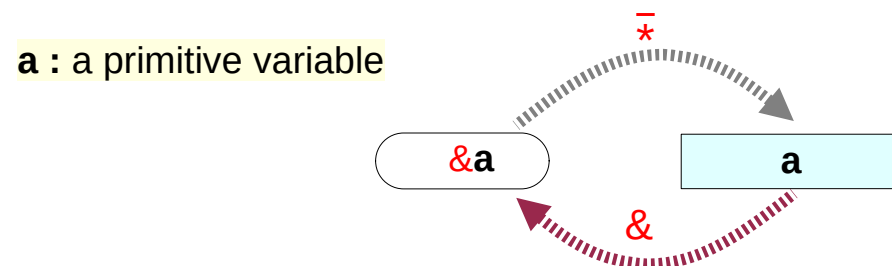
X : a variable `*p`, `p`, `a`



Pointer Dereferencing



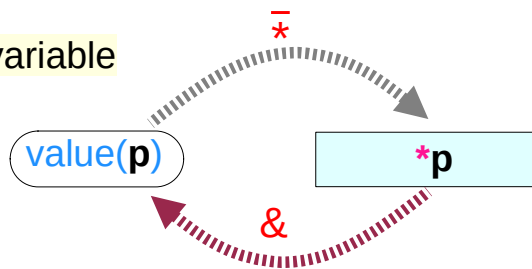
Pointer Data



Non-pointer Data

Inverse relations of $\&$ and $\bar{*}$ operators

$*p$: a dereferenced variable

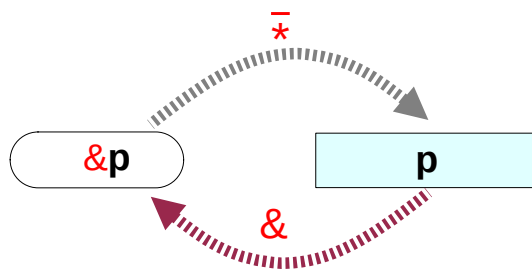


$$\bar{*} \text{value}(p) = *p$$

$$\&\bar{*} \text{value}(p) = \&*p = \text{value}(p)$$

$$\bar{*}\&\bar{*} \text{value}(p) = \bar{*}\&*p = \bar{*} \text{value}(p) = *p$$

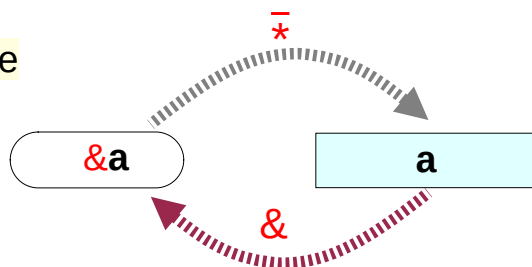
p : a pointer variable



$$\bar{*}\&p = p$$

$$\&\bar{*}\&p = \&p$$

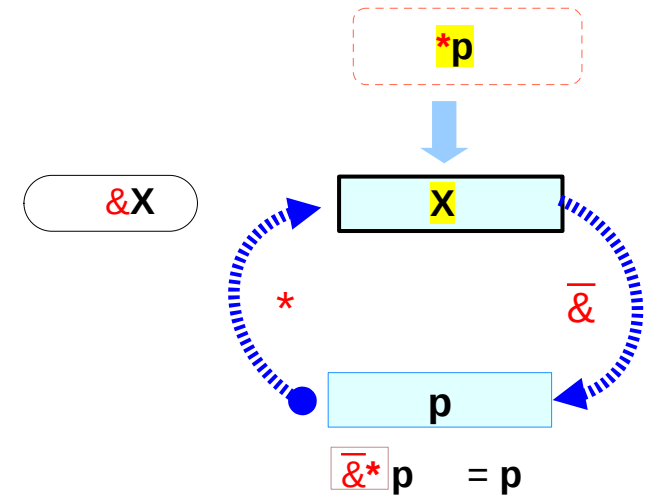
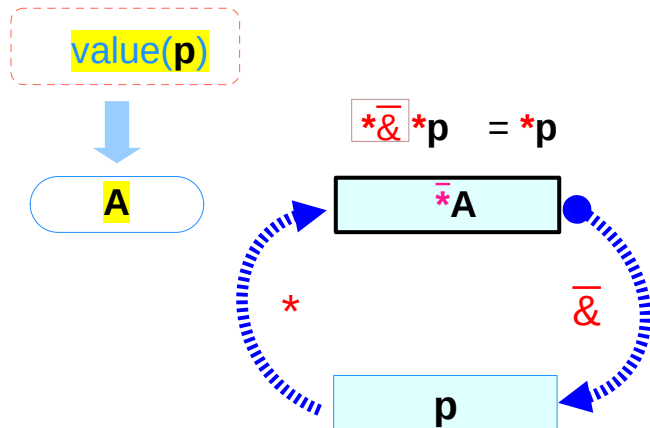
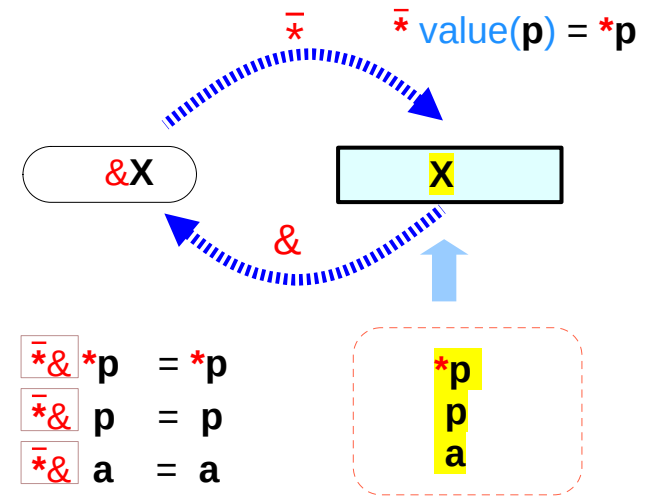
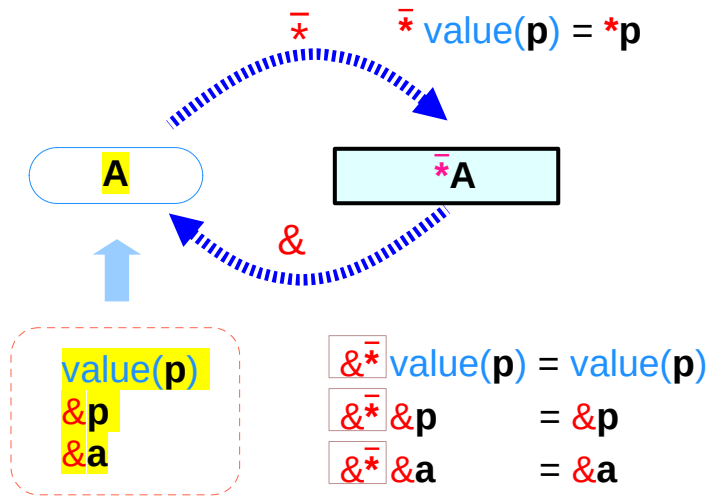
a : a primitive variable



$$\bar{*}\&a = a$$

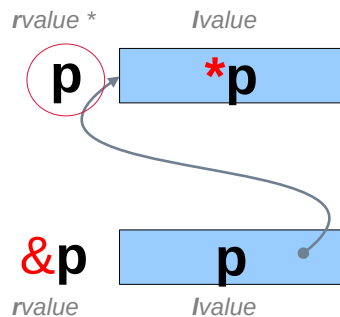
$$\&\bar{*}\&a = \&a$$

Requirements of pointed address and data

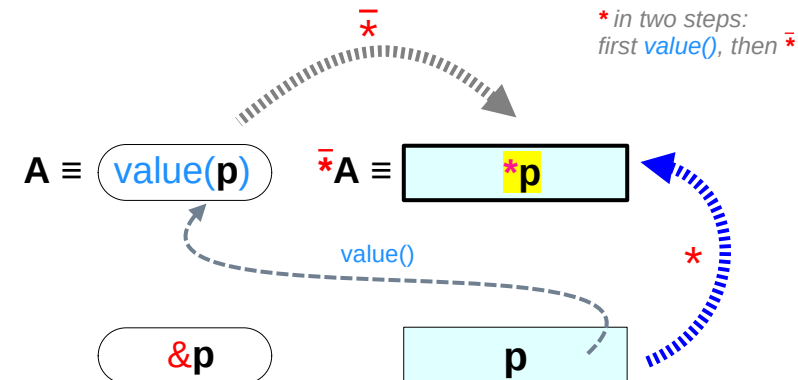


& and * operators in pointer de-referencing (1)

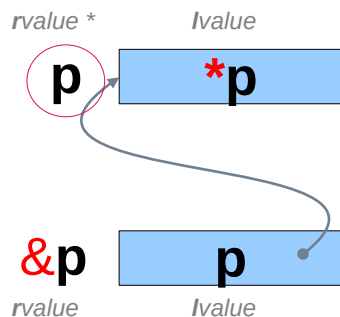
Two step De-reference * operation



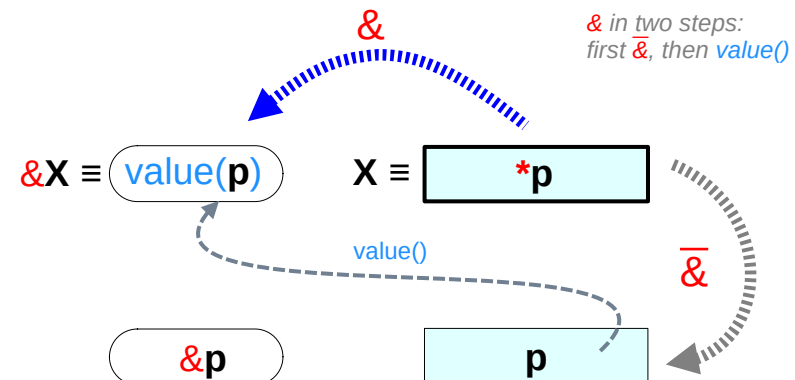
$$\begin{array}{lcl} \boxed{*p} & = & \boxed{\bar{*}\text{value}(p)} \\ \text{C Expressions} & & \text{Mixed Expressions} \\ \boxed{\bar{*}A} & = & \boxed{* \text{value}^{-1}(A)} \end{array}$$



Two step Address-of & operation

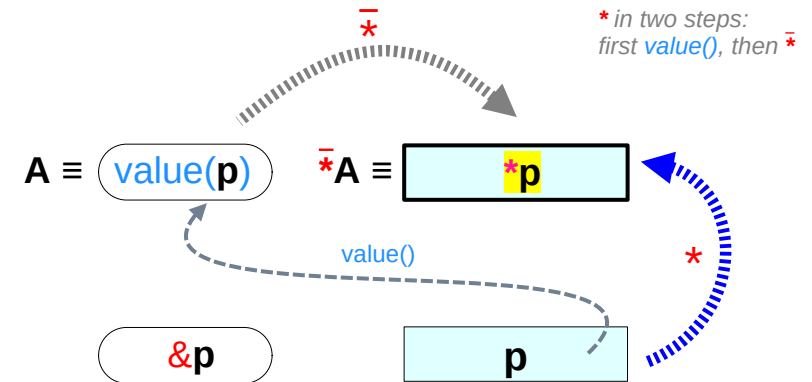
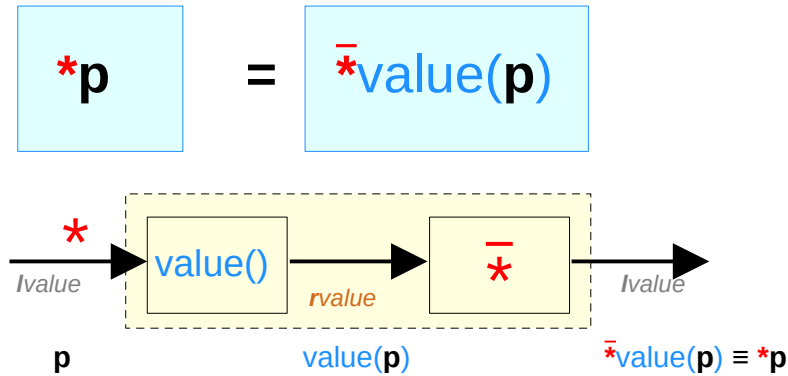


$$\begin{array}{lcl} \boxed{\&X} & = & \boxed{\text{value}(\bar{\&X})} \\ \text{C Expressions} & & \text{Mixed Expressions} \\ \boxed{\bar{\&X}} & = & \boxed{\text{value}^{-1}(\&X)} \end{array}$$

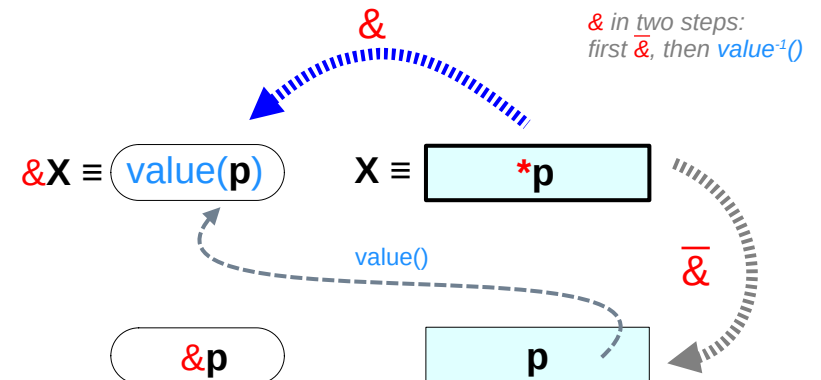
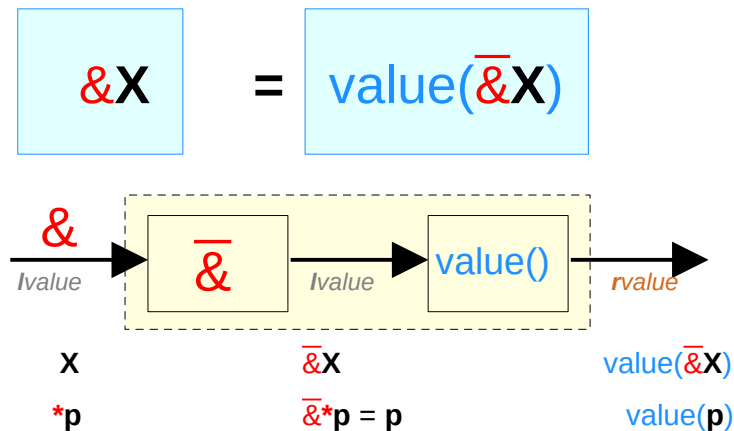


& and * operators in pointer de-referencing (2)

Two step De-reference * operation

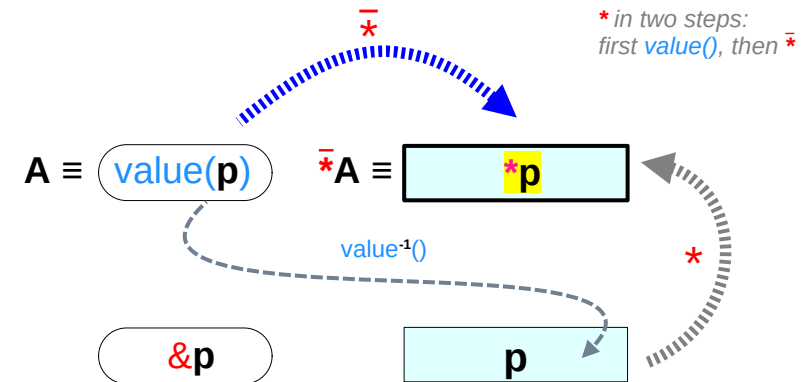
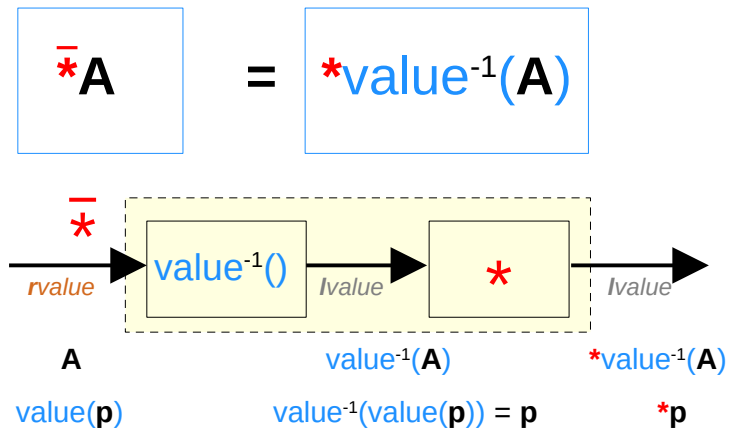


Two step Address-of & operation

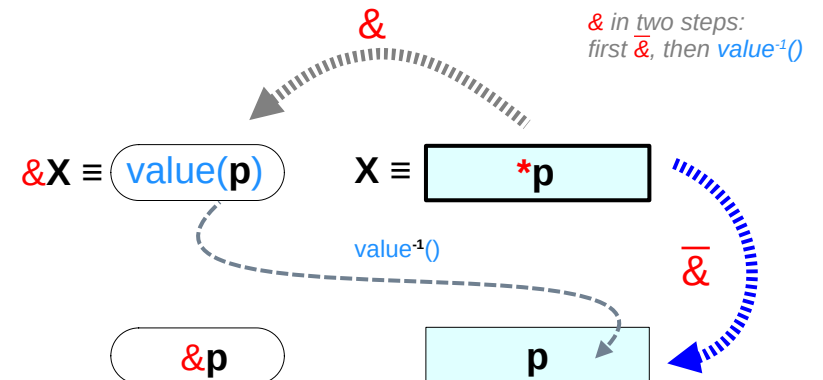
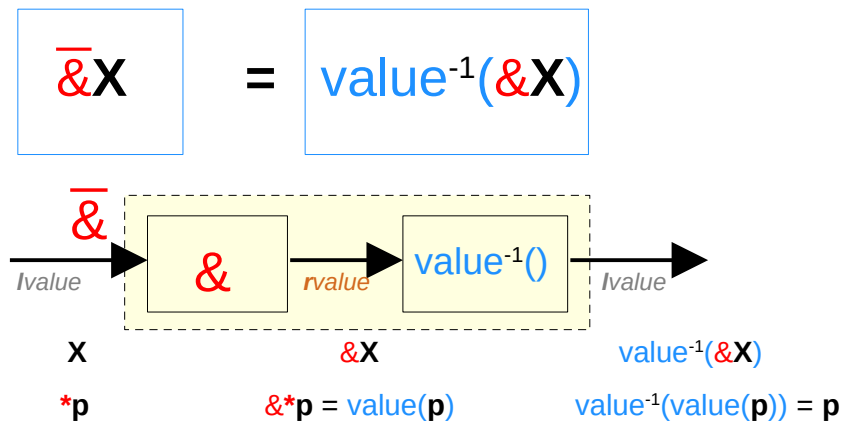


$\bar{*}$ and $\bar{*}$ operators in pointer de-referencing (3)

Two step De-reference $\bar{*}$ operation



Two step Address-of $\bar{\&}$ operation



Two step address-of & and $\bar{\&}$ operators

for $\&X$, X can be $*p$, p , a

$$\&X = \text{value}(\bar{\&X})$$

C Expressions

for $\bar{\&X} = p$, X must be $*p$

$$\bar{\&X} = \text{value}^{-1}(\&X)$$

$*p$ a dereferenced variable
 p a pointer variable
 a a primitive variable

$\&X$

$$\&*p = \&* \text{value}(p) = \text{value}(p)$$

$\&p$

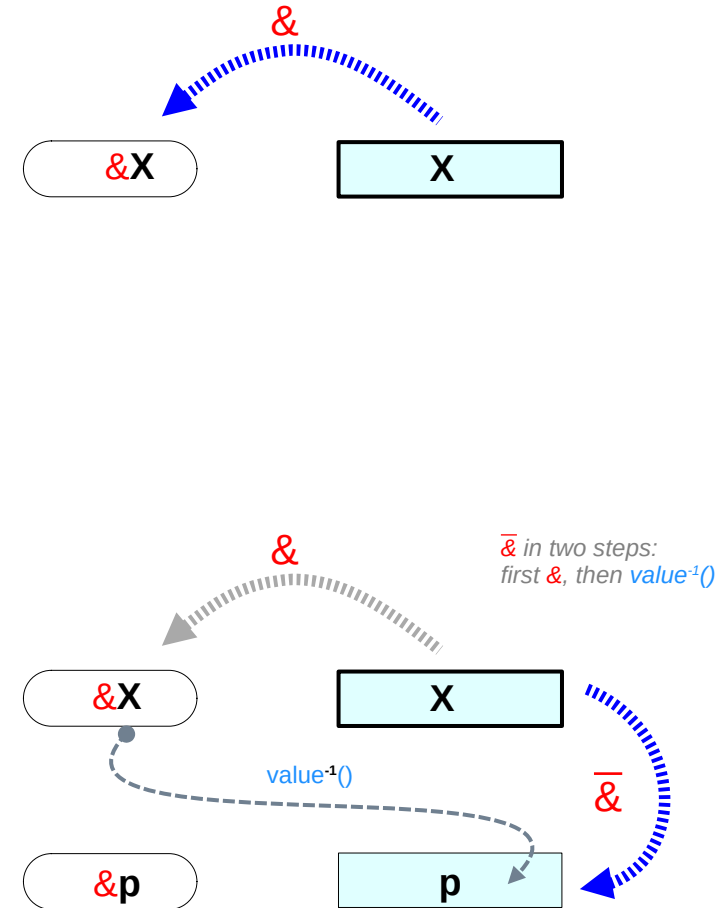
$\&a$

$*p$ a dereferenced variable
 ~~p a pointer variable~~
 ~~a a primitive variable~~

$$\bar{\&X} = p$$

$$\bar{\&*p} \equiv p$$

~~$\bar{\&p}$~~
 ~~$\bar{\&a}$~~



Two step de-reference $\bar{*}$ and $*$ operators

for $\bar{*}A$, A can be $\text{value}(p)$, $\&p$, $\&a$

$\text{value}(p)$ value of a pointer variable
 $\&p$ address of a pointer variable
 $\&a$ address of a primitive variable

$$\bar{*}A = * \text{value}^{-1}(A)$$

$$\begin{aligned} \bar{*}A \\ \bar{*} \text{value}(p) &= *p \\ \bar{*} \&p &= p \\ \bar{*} \&a &= a \end{aligned}$$

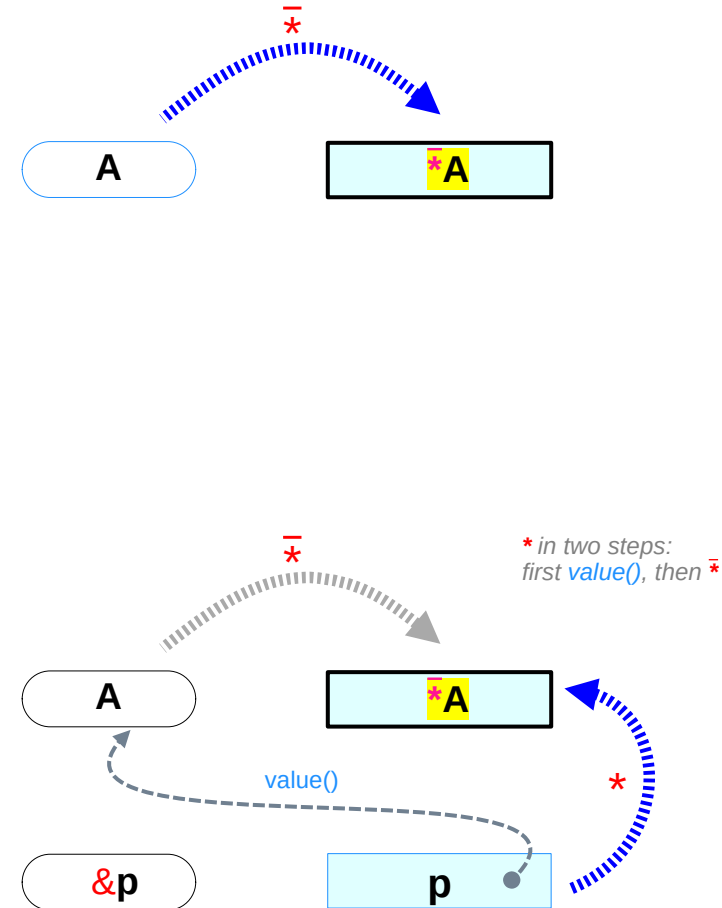
for $\bar{*}A = *p$, A must be $\text{value}(p)$

$\text{value}(p)$ value of a pointer variable
 ~~$\&p$ address of a pointer variable~~
 ~~$\&a$ address of a primitive variable~~

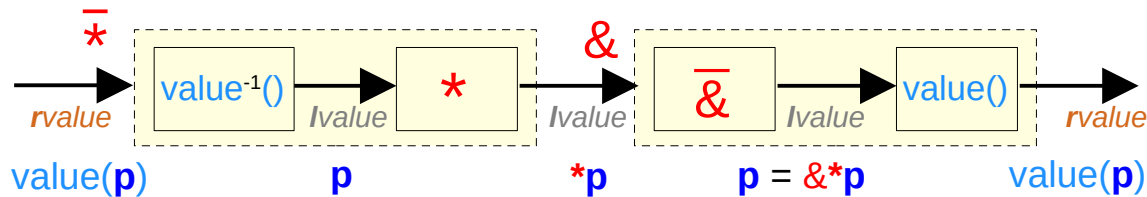
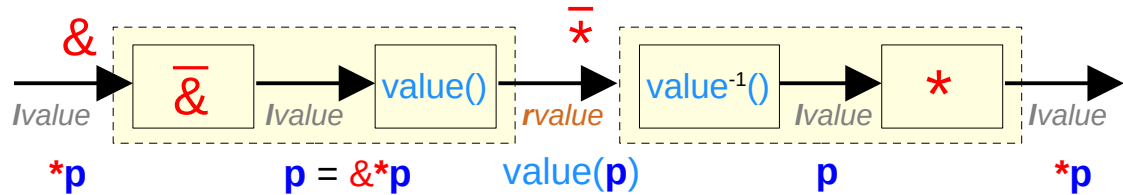
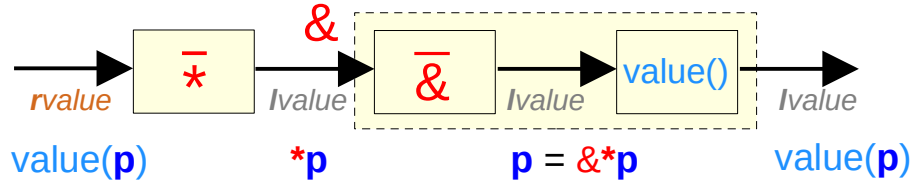
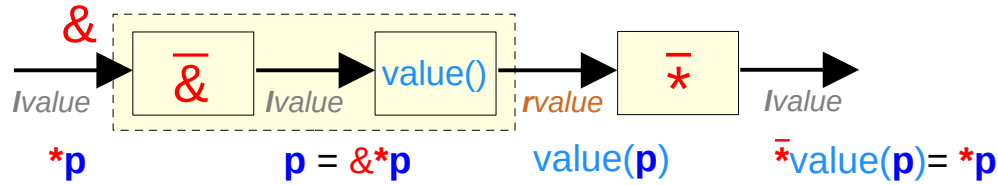
$$*p = \bar{*} \text{value}(p)$$

C Expressions

$$\begin{aligned} *p \\ *p &\equiv \bar{*} \text{value}(p) \\ \cancel{* \&p} &= p \\ \cancel{* \&a} &= a \end{aligned}$$



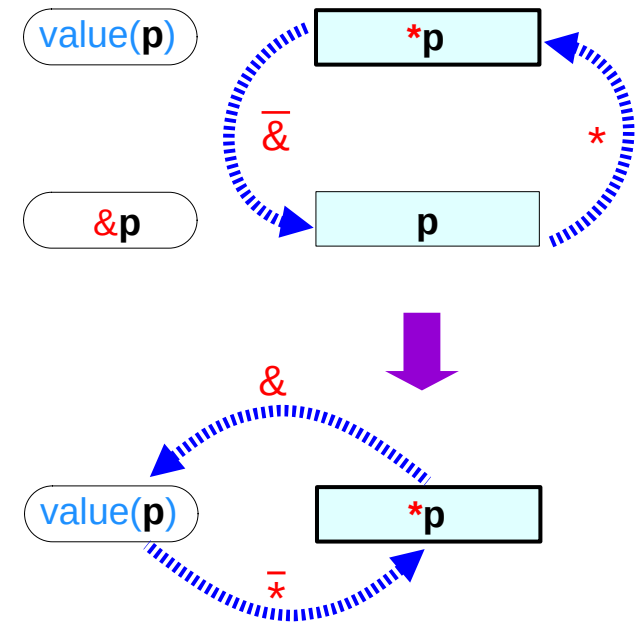
Inverse operators : $\&$ and $\bar{*}$ in pointer de-referencing



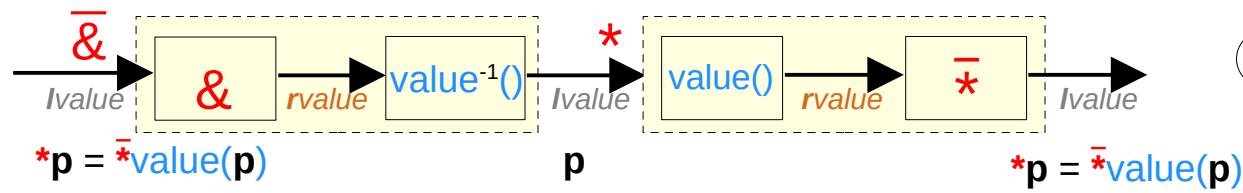
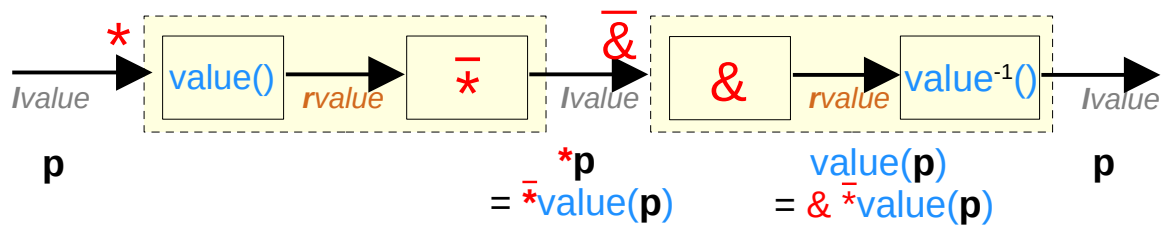
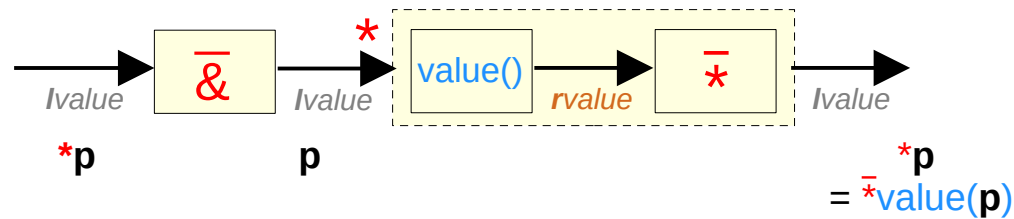
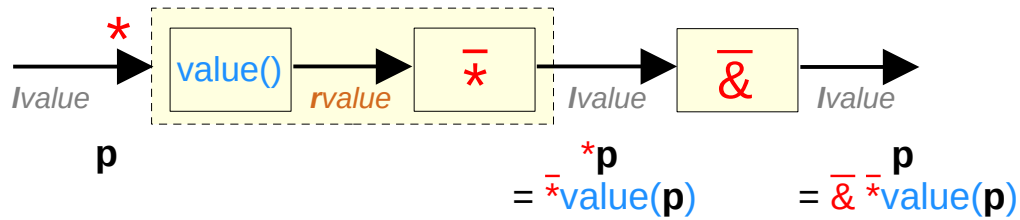
$$\bar{*}value(p) = *p$$

$$\&*value(p) = \&*p = value(p)$$

$$*\&*value(p) = *\&*p = *\&*p = *p$$



Inverse operators of & and *



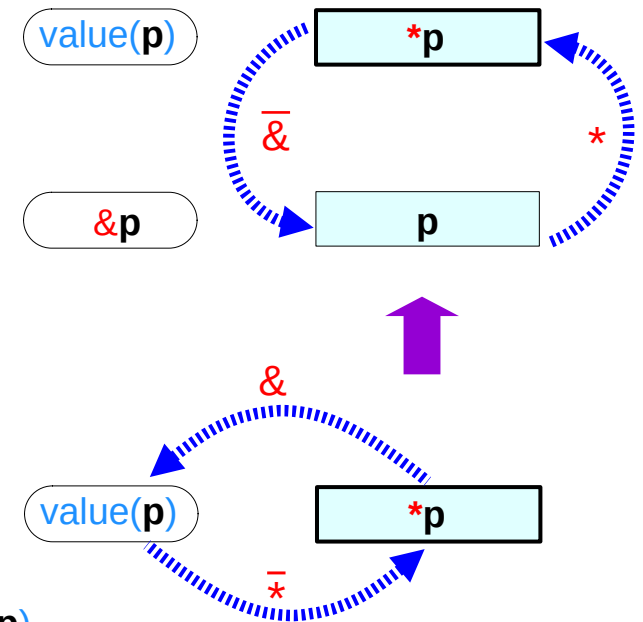
$$\&* \text{value}(p) = *p$$

$$\&* p = p$$

$$*\&p = p$$

$$p = \&* \text{value}(p)$$

$$\text{value}(p) = \&* \text{value}(p)$$



Inverse operators in pointer referencing (1)

$$\begin{aligned}\overline{*}&\&*p &= *p \\ \overline{*}&\&p &= p \\ \overline{*}&\&a &= a\end{aligned}$$

$$\begin{aligned}\&\overline{*} \text{value}(p) &= \text{value}(p) \\ \&\overline{*} \&p &= \&p \\ \&\overline{*} \&a &= \&a\end{aligned}$$

$\overline{*}&$

$*p$
 p
 a

$\overline{\&*}$

p

$\&\overline{*}$

$\text{value}(p)$
 $\&p$
 $\&x$

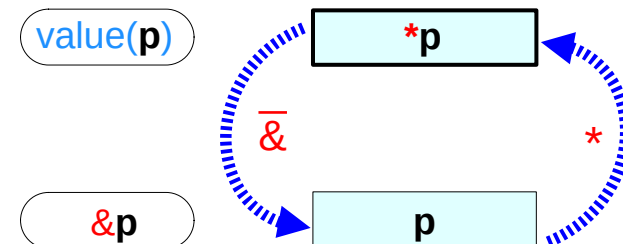
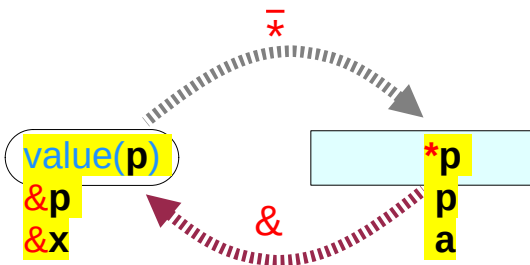
$*\overline{\&}$

$*p$

$$\overline{\&*} p = p$$

$$\&\overline{*} *p = *p$$

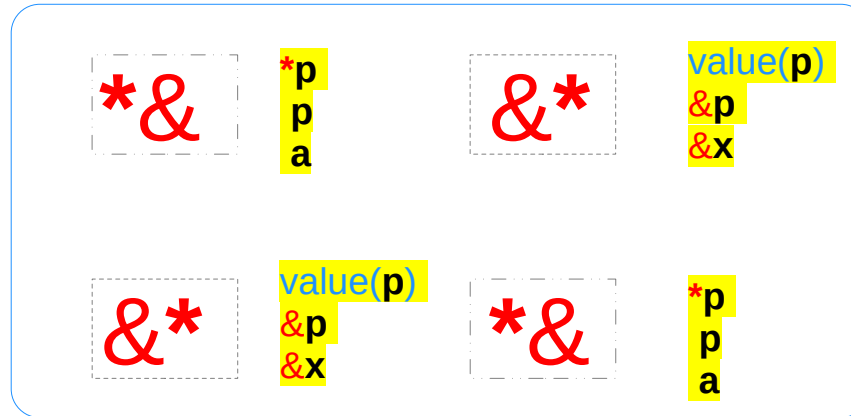
Math expressions



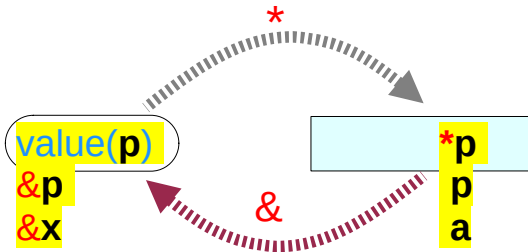
Inverse operators in pointer referencing (2)

`*&*p = *p`
`*&p = p`
`*&a = a`

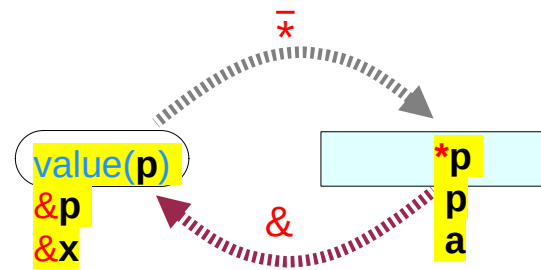
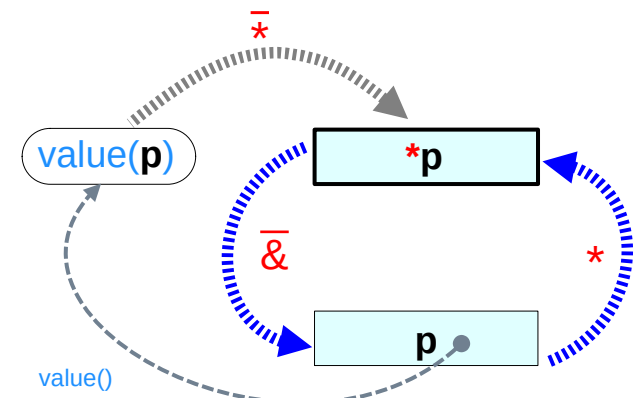
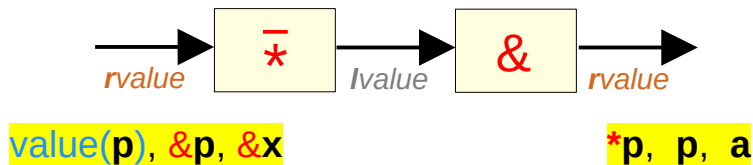
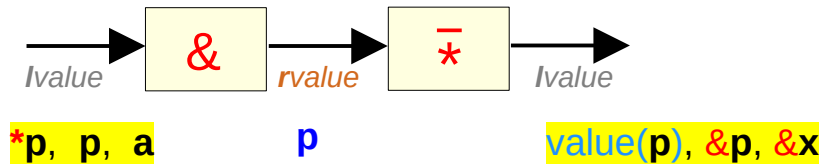
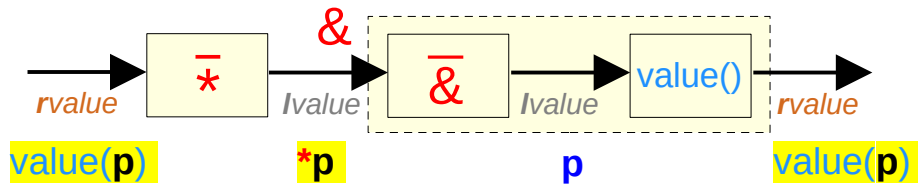
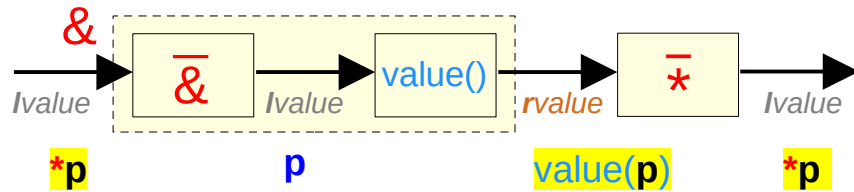
`&*value(p) = value(p)`
`&*&p = &p`
`&*&a = &a`



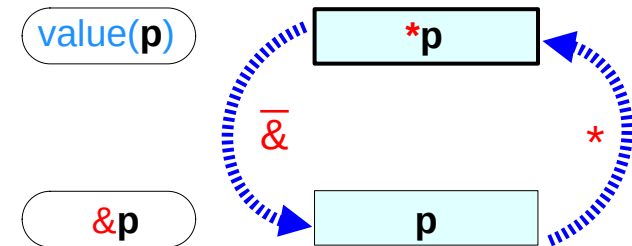
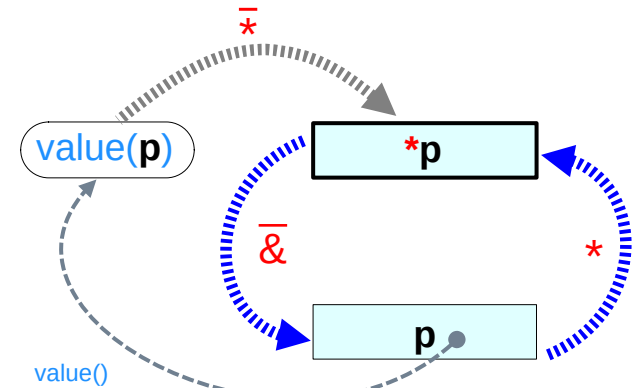
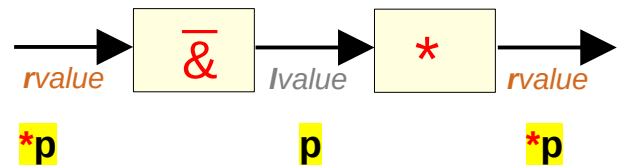
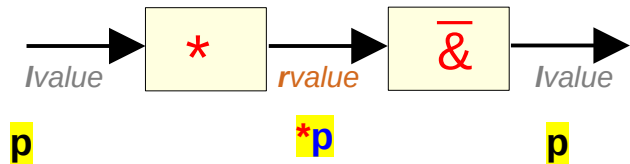
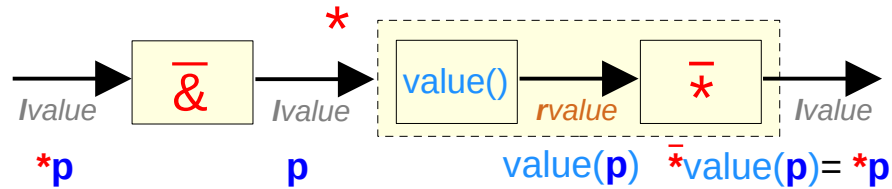
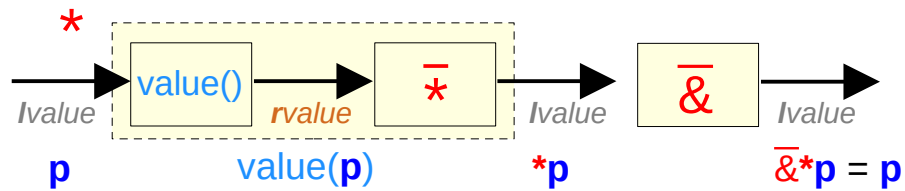
C expressions



Inverse operators in pointer referencing (3)



Inverse operators in pointer referencing (3)



References

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- [2] Efficient C Programming, Mark A. Weiss
- [3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
- [4] C Language Express, I. K. Chun